

الف) $y = x^3 - 3x^2 + 3x$

$\Rightarrow y = \frac{1}{x^2 - 2x}$

$y = (x-1)^3 + 1$

$y = \frac{1}{x(x-2)}$ $x \neq 0$
 $x \neq 2$

$R_y = R$

$R_y = R$

$0 < y < 1$
 $0 > y > 2$
 $0 < y < 2$

الف) $y = x^2 - 4x + 10$

$\begin{cases} x_{min} = \frac{-b}{2a} = \frac{4}{2} = 2 \\ y_{min} = 2 \end{cases}$

$R_y = [2, +\infty)$

$\Rightarrow y = -x^2 + 4x + 13$

$\begin{cases} x_{max} = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y_{max} = 17 \end{cases}$

$R_y = (-\infty, 17]$

ج) $y = \sqrt{x^2 - 4x - 3}$

$\begin{cases} x_{min} = \frac{-b}{2a} = \frac{4}{2} = 2 \\ y_{min} = -1 \end{cases}$

$R_y = [0, +\infty)$

$x^2 - 4x - 3 \geq 0$

$\Rightarrow y = \sqrt{4x - 2x}$

$\begin{cases} x_{max} = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y_{max} = 2 \end{cases}$

$\Rightarrow R_y = \sqrt{(-\infty, 4]} \Rightarrow [0, 2]$

$x \geq \sqrt{4x-2x}$ $x \leq \sqrt{4x-2x}$

الف) $y = x^3 - 2x^2 + 2x + 1$ $R_y = R$

$\Rightarrow y = x^3 - 4x^2 + 3x + 2$ $R_y = R$

ج) $y = \sqrt{x^3 - 4x^2 + 3x + 1}$ $R_y = [0, +\infty)$

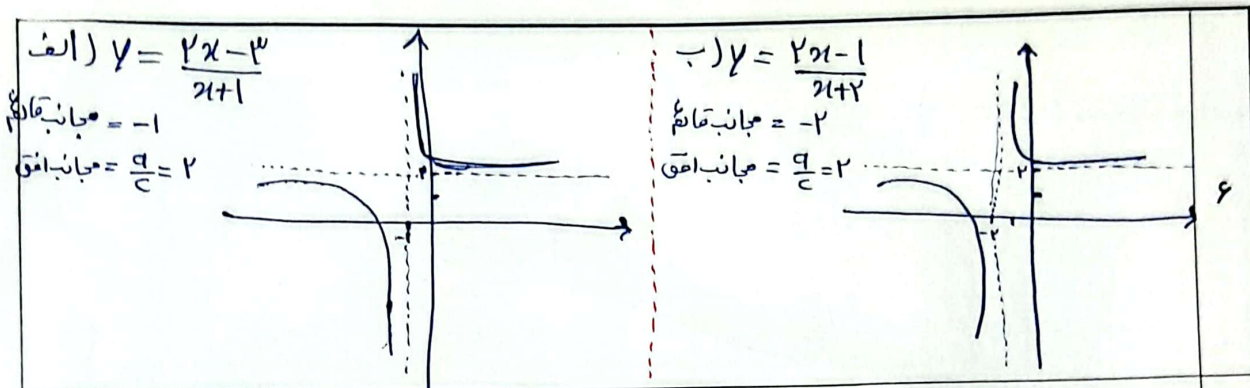
د) $y = (x^3 - 4x^2 + 3x + 1)^2$ $R_y = [0, +\infty)$

الف) $y = \frac{x+1}{x-2} \Rightarrow R_y = R - \{2\}$
 $\frac{a}{c} = 1$

$\Rightarrow y = \frac{x+1}{x+1} \Rightarrow R_y = R - \{1\}$
 $\frac{a}{c} = 1$

الف) $y = \sqrt{\frac{x+1}{x+1}} \Rightarrow R_y = \sqrt{R - \{1\}} \Rightarrow R_y = [0, +\infty) - \{\sqrt{1}\}$
 $\frac{a}{c} = 1$

ب) $y = \sqrt{\frac{x+1}{1-x}} \Rightarrow R_y = \sqrt{R - \{1\}} \Rightarrow R_y = [0, +\infty)$
 $\frac{a}{c} = -1$



الف) $y = \sinh x + \frac{1}{\sinh x}$ $Ry = (-\infty, -2] \cup [2, +\infty)$

ب) $y = \frac{x^4+1}{x^2} \Rightarrow x^2 + \frac{1}{x^2} \Rightarrow Ry = (-\infty, -2] \cup [2, +\infty)$

ج) $y = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$ $Ry = (-\infty, -2] \cup [2, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ $Ry = [2, +\infty)$

الف) $y = x^2 + \frac{1}{x^2+3}$

$\Rightarrow x^2 = 0$

$y_{\min} = \frac{1}{3}$

$Ry = [\frac{1}{3}, +\infty)$

ب) $y = \frac{x^2+\omega}{\sqrt{x^2+\epsilon}} \Rightarrow x^2 = 0$
 $y_{\min} = \frac{\omega}{\sqrt{\epsilon}}$

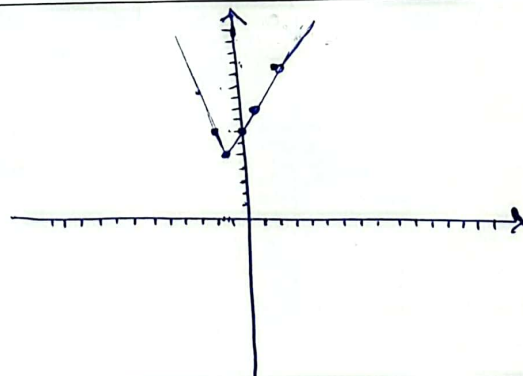
$\frac{\omega}{\sqrt{\epsilon}} = \frac{\omega}{\sqrt{\epsilon}}$

$Ry = [\frac{\omega}{\sqrt{\epsilon}}, +\infty)$

$y = |x-3| + |3x+5|$

$$\begin{cases} x-3+3x+5 & x > 3 \\ -x+3+3x+5 & -\frac{5}{3} \leq x \leq 3 \\ -x+3-3x-5 & x < -\frac{5}{3} \end{cases}$$

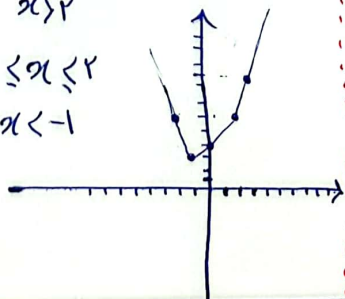
$Ry = [\omega, +\infty)$



الف) $y = |x-2| + |2x+2|$

$$\begin{cases} x-2+2x+2 & x > 2 \\ -x+2+2x+2 & -1 \leq x \leq 2 \\ -x+2-2x-2 & x < -1 \end{cases}$$

$Ry = [2, +\infty)$



ب) $y = |x-2| - |x+1|$

$$\begin{cases} x-2-x-1 & x > 1 \\ -x-2-x-1 & -1 \leq x \leq 1 \\ -x-2+x+1 & x < -1 \end{cases}$$

$Ry = [-2, +\infty)$

