

الف) $y = \frac{n^3 - 4n^2 + 4n}{n^2 - 4n + 4} = \frac{n^3 - 4n^2 + 4n - 1 + 1}{n^2 - 4n + 4} = \frac{(n-1)^3 + 1}{n^2 - 4n + 4} = y$
 $n-1 = \sqrt[3]{y-1} \rightarrow n = \sqrt[3]{y-1} + 1 \rightarrow R_f = \mathbb{R}$

ب) $y = \frac{1}{n^2 - 4n} \rightarrow y(n^2 - 4n) = 1 \rightarrow y(n^2 - 4n) = 1 \rightarrow n^2 - 4n = \frac{1}{y}$
 $\rightarrow n^2 - 4n + 1 = \frac{1}{y} + 1 \rightarrow (n-1)^2 = \frac{1}{y} + 1 \rightarrow n-1 = \pm \sqrt{\frac{1}{y} + 1} \rightarrow n = \pm \sqrt{\frac{1}{y} + 1} + 1$

الف) $y = 2^x - \varepsilon n + 1 \rightarrow a > 0 \rightarrow \min$
 $S \mid -\frac{b}{a} = \frac{\varepsilon}{2} = 2 \rightarrow R_f = [4, +\infty)$

ب) $y = -n^2 + 4n + 4 \rightarrow a < 0 \rightarrow \max$
 $S \mid -\frac{b}{a} = \frac{4}{-2} = -2 \rightarrow R_f = (-\infty, 12]$

ج) $y = \sqrt{n^2 - \varepsilon n - 2} \rightarrow a > 0 \rightarrow \min$
 $S \mid -\frac{b}{a} = \frac{\varepsilon}{2} = 2 \rightarrow R_f = [0, +\infty)$

د) $y = \sqrt{4n - n^2} \rightarrow a < 0 \rightarrow \max$
 $S \mid -\frac{b}{a} = \frac{4}{-1} = -4 \rightarrow R_f = [0, 2]$

الف) $y = n^3 - 4n^2 + 4n + 1 \rightarrow R_f = \mathbb{R}$

ب) $y = n^3 - 4n^2 + 4n + 2 \rightarrow R_f = \mathbb{R}$

ج) $y = \sqrt{n^3 - 4n^2 + 4n + 1} \rightarrow R_f = [0, +\infty)$

د) $y = (n^3 - 4n^2 + 4n + 1)^2 \rightarrow R_f = [0, +\infty)$

الف) $y = \frac{3n+1}{n-2} \rightarrow R_f = \mathbb{R} - \{3\}$

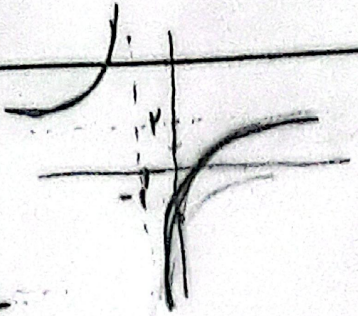
ب) $y = \frac{3n+1}{n+1} \rightarrow R_f = \mathbb{R} - \{3\}$

الف) $y = \sqrt{\frac{3n+1}{n+1}} \rightarrow R_f = [0, +\infty) - \{\sqrt{2}\}$

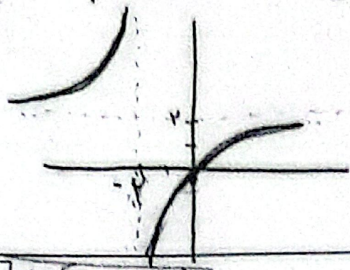
ب) $y = \sqrt{\frac{3n+1}{3-n}} \rightarrow R_f = [0, +\infty)$

الف) $y = \frac{x-3}{x+1}$ $\xrightarrow{\text{مجاوب افقی: } y=1}$ $\rightarrow m+1=0 \rightarrow m=-1$

$\downarrow$
 $x=0 \rightarrow y=-3$ $\text{مجاوب افقی: } \frac{a}{c} = \frac{1}{1} = 1$



ب) $y = \frac{x-1}{x+2}$ $\xrightarrow{\text{مجاوب افقی: } y=-1}$
 $\xrightarrow{\text{مجاوب افقی: } y=2}$



$m=0 \rightarrow y = \frac{1}{2}$

الف) $y = \sin x + \frac{1}{\sin x}$ $(R_f = (-\infty, -2] \cup [2, +\infty))$

ب) $y = \frac{x^4+1}{x^2} = x^2 + \frac{1}{x^2} \rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

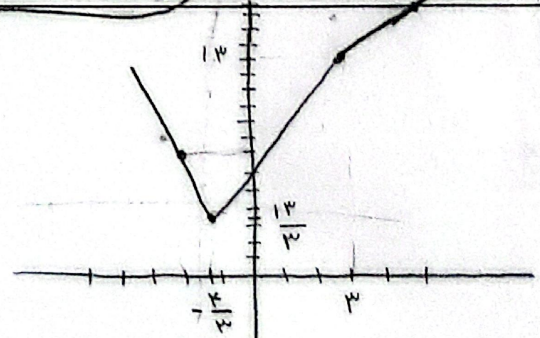
ج) $y = \frac{\sqrt{x^2+1}}{\sqrt{x}} = \sqrt{x} + \frac{1}{\sqrt{x}} \rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \rightarrow R_f = [2, +\infty)$

الف) $y = x^2 + \frac{1}{x^2+3} + 3 - 3 = x^2 + 3 + \frac{1}{x^2+3} - 3 \rightarrow$ $\text{کمترین حالت } x^2+3 = 3$
 $x=0$ است $R_f = [3 + \frac{1}{3}, +\infty) \rightarrow R_f = [\frac{10}{3}, +\infty)$

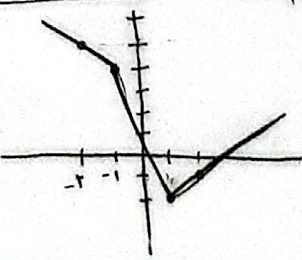
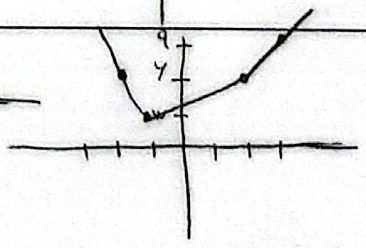
ب) $y = \frac{x^2+a}{\sqrt{x^2+\varepsilon}} = \frac{x^2+\varepsilon+1}{\sqrt{x^2+\varepsilon}} = \sqrt{x^2+\varepsilon} + \frac{1}{\sqrt{x^2+\varepsilon}} \rightarrow$ $\text{کمترین حالت } x^2+\varepsilon = \varepsilon$
 $x=0$ است $\sqrt{x^2+\varepsilon} = \sqrt{\varepsilon}$ است $R_f = [\varepsilon + \frac{1}{\sqrt{\varepsilon}}, +\infty) \rightarrow R_f = [\frac{a}{\sqrt{\varepsilon}} + \frac{1}{\sqrt{\varepsilon}}, +\infty)$

$y = \frac{|x-3|}{x} + \frac{x^2+\varepsilon}{x}$
 $\frac{x-3}{x} + \frac{x^2+\varepsilon}{x} = \frac{x-3+x^2+\varepsilon}{x} = \frac{x^2+x-3+\varepsilon}{x}$
 $\frac{x^2+x-3+\varepsilon}{x} = x + 1 - \frac{3-\varepsilon}{x}$
 $x = \frac{3-\varepsilon}{2} \rightarrow y = \frac{1}{2}$
 $x = 3 \rightarrow y = 1$



الف) $y = |x-2| + |x+2|$
 $x \geq 2 \rightarrow y = 4$
 $x = -1 \rightarrow y = 3$
 $x = 3 \rightarrow y = 4$
 $x = -2 \rightarrow y = 4$
 ب) $y = |x-2| - |x+1|$
 $x \geq 1 \rightarrow y = -2$
 $x = -1 \rightarrow y = 3$
 $x = 2 \rightarrow y = 2 - 3 = -1$
 $x = -2 \rightarrow y = 4 - 1 = 3$

$R_f = [3, +\infty)$



$R_f = [-2, +\infty)$

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