

الف) $y = \frac{n^3 - 4n^2 + 4n}{n^2 - 4n + 4} = \frac{n^3 - 4n^2 + 4n - 1 + 1}{n^2 - 4n + 4} = \frac{(n-1)^3 + 1}{n^2 - 4n + 4} = y$
 $n-1 = \sqrt[3]{y-1} \rightarrow n = \sqrt[3]{y-1} + 1 \rightarrow R_f = \mathbb{R}$

$R_f = (-\infty, -1] \cup (0, +\infty)$

ب) $y = \frac{1}{n^2 - 4n} \rightarrow y(n^2 - 4n) = 1 \rightarrow y(n^2 - 4n) = 1 \rightarrow n^2 - 4n = \frac{1}{y}$
 $\rightarrow n^2 - 4n + 1 = \frac{1}{y} + 1 \rightarrow (n-2)^2 = \frac{1}{y} + 1 \rightarrow n-2 = \pm \sqrt{\frac{1}{y} + 1} \rightarrow n = 2 \pm \sqrt{\frac{1}{y} + 1}$

الف) $y = 2^x - \varepsilon n + 1 \rightarrow a > 0 \rightarrow \min$
 $S \mid -\frac{b}{a} = \frac{\varepsilon}{2} = 2 \rightarrow R_f = [4, +\infty)$

ب) $y = -n^2 + 4n + 4 \rightarrow a < 0 \rightarrow \max$
 $S \mid -\frac{b}{a} = \frac{4}{-2} = -2 \rightarrow R_f = (-\infty, 12]$

ج) $y = \sqrt{n^2 - \varepsilon n - 2} \rightarrow a > 0 \rightarrow \min$
 $S \mid -\frac{b}{a} = \frac{\varepsilon}{2} = 2 \rightarrow R_f = [0, +\infty)$

د) $y = \sqrt{4n - n^2} \rightarrow a < 0 \rightarrow \max$
 $S \mid -\frac{b}{a} = \frac{4}{-2} = -2 \rightarrow R_f = [0, 2]$

الف) $y = n^3 - 4n^2 + 4n + 1 \rightarrow R_f = \mathbb{R}$

ب) $y = n^3 - 4n^2 + 4n + 2 \rightarrow R_f = \mathbb{R}$

ج) $y = \sqrt{n^3 - 4n^2 + 4n + 1} \rightarrow R_f = [0, +\infty)$

د) $y = (n^3 - 4n^2 + 4n + 1)^2 \rightarrow R_f = [0, +\infty)$

الف) $y = \frac{3n+1}{n-2} \rightarrow R_f = \mathbb{R} - \{2\}$

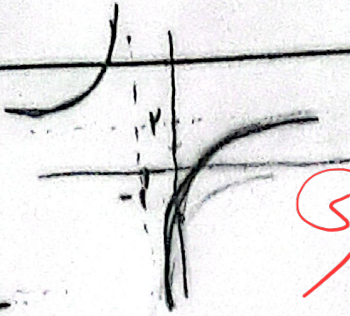
ب) $y = \frac{3n+1}{n+1} \rightarrow R_f = \mathbb{R} - \{2\}$

الف) $y = \sqrt{\frac{3n+1}{n+1}} \rightarrow R_f = [0, +\infty) - \{\sqrt{2}\}$

ب) $y = \sqrt{\frac{3n+1}{3-n}} = \sqrt{\frac{3n+1}{-n+3}} \rightarrow R_f = [0, +\infty)$

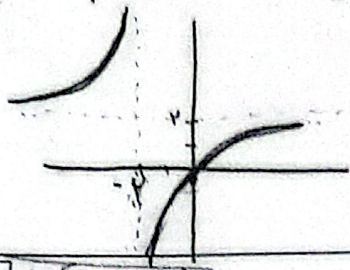
الف) $y = \frac{x-3}{x+1}$ $\xrightarrow{\text{مجاوبه}} \rightarrow m+1=0 \Rightarrow m=-1$

$\downarrow$
 $x=0 \Rightarrow y = -3$ $\xrightarrow{\text{مجاوبه}} \rightarrow \frac{a}{c} = \frac{1}{1} = 1$



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ب) $y = \frac{x-1}{x+2}$ $\xrightarrow{\text{مجاوبه}} \rightarrow -2$
 $\xrightarrow{\text{مجاوبه}} \rightarrow 2$



$m = \infty \Rightarrow y = \frac{1}{2}$

الف) $y = \sin x + \frac{1}{\sin x}$ $(R_f = (-\infty, -2] \cup [2, +\infty))$

ب) $y = \frac{x^4+1}{x^2} = x^2 + \frac{1}{x^2} \rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

ج) $y = \frac{\sqrt{x^2+1}}{\sqrt{x}} = \sqrt{x} + \frac{1}{\sqrt{x}} \rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \rightarrow R_f = [2, +\infty)$

الف) $y = x^2 + \frac{1}{x^2+3} + 3 - 3 = x^2 + 3 + \frac{1}{x^2+3} - 3 \rightarrow x^2 + 3$
 $R_f = [3 + \frac{1}{4} - 3, +\infty) \rightarrow R_f = [\frac{1}{4}, +\infty)$
 استقره $a=3$

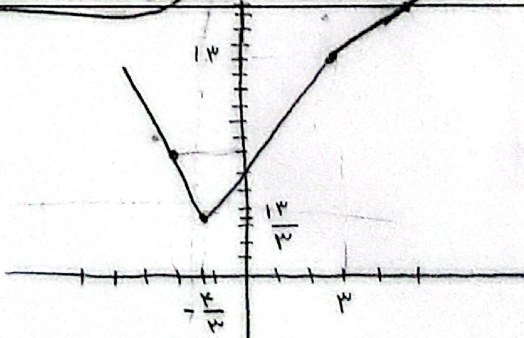
ب) $y = \frac{x^2+a}{\sqrt{x^2+2}} = \frac{x^2+\varepsilon+1}{\sqrt{x^2+2}} = \sqrt{x^2+2} + \frac{1}{\sqrt{x^2+2}}$
 $R_f = [2 + \frac{1}{2}, +\infty) \rightarrow R_f = [\frac{5}{2}, +\infty)$
 استقره $a=2$
 استقره $\sqrt{x^2+2} = 2$

$y = \frac{|x-3|}{x} + \frac{x^2+2}{x}$

$\frac{x-3}{x} + \frac{x^2+2}{x} = \frac{x-3+x^2+2}{x} = \frac{x^2+x-1}{x}$

$x=0 \Rightarrow y = \frac{1}{x}$

$x=3 \rightarrow y=13$



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الف) $y = |x-2| + |x+2|$ $(R_f = [4, +\infty))$

$x=2 \rightarrow y=4$

$x=-1 \rightarrow y=3$

$x=3 \rightarrow y=4$

ب) $y = |x-2| - |x+2|$

$x=1 \rightarrow y=-2$

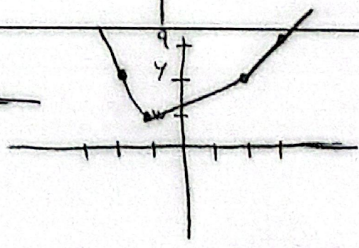
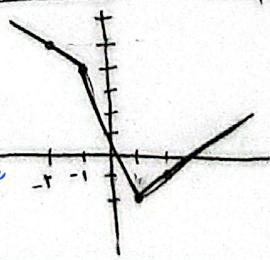
$x=-1 \rightarrow y=2$

$x=2 \rightarrow y=2-4=-2$

$x=-2 \rightarrow y=4-1=3$

$\frac{1}{x-2} + \frac{1}{x+2}$

$\frac{1}{x-2} - \frac{1}{x+2}$



$R_f = [-2, +\infty)$

مجاوبه

1,0