گروه دهم لم B — حسابان ۲ تشریحی — ۲۰

① ۶

الف) $y=x^3-3x^2+3x \to (x-1)^3+1 \to y=(x-1)^3+1$

$y-1=(x-1)^3 \to \sqrt[3]{y-1}=(x-1) \to \sqrt[3]{y-1}+1=x$

$x=\sqrt[3]{y-1}+1 \to R_f$ اعداد R

ب) $y=\frac{1}{x^2-2x} \to y(x^2-2x)=1 \to x^2-2x=\frac{1}{y}$

$x^2-2x+1=\frac{1}{y}+1 \to (x-1)^2=\frac{1}{y}+1 \to x-1=\pm\sqrt{\frac{1}{y}+1}$

$x=\pm\sqrt{\frac{1}{y}+1}+1 \to \frac{1}{y}+1\geq 0 \to \frac{1+y}{y}\geq 0$; $1+y=0 \to y=-1$, $y=0$

	-1		0	
1+y	−	+		+
y	−	−		+
	+	−		+

$\to R_f=(-\infty,-1]\cup(0,+\infty)$

② ۶

الف) $y=x^2-4x+10 \to x_{min}=\frac{-b}{2a} \to \frac{4}{2}=2 \to y_{min}=4-8+10=6$

$R_f=[6,+\infty)$

ب) $y=-x^2+6x+3 \to x_{max}=\frac{-b}{2a} \to \frac{-6}{-2}=3 \to y_{max}=12$

$R_f=[-\infty,12]$

ج) $y=\sqrt{x^2-4x-3} \to x_{min}=\frac{4}{2}=2 \to y_{min}=-7 \to [-7,+\infty) \to [0,+\infty)$

د) $y=\sqrt{6x-x^2} \to -x^2+6x \to x_{max}=\frac{-6}{-2}=3 \to y_{max}=9$

$\to [0,3]$

ب) $y = x^3 - 3x^2 + 2x + 1 \to \mathbb{R}$

ج) $y = x^4 - 4x^3 + 2x + 1 \to \mathbb{R}$

د) $y = \sqrt{x^2 - 4x + 3} + 1 \to [0, +\infty)$

هـ) $y = (x^2 - 2x^2 + 2x + 1)^2 \to R_f = [0, +\infty)$

و) $y = \frac{2x+1}{x-1} \to \frac{ax+b}{cx+d}$ $ad \neq cb$ ✓

$R_f = R - [2]$

$\mathbb{R} - \left[\frac{a}{c}\right]$

ز) $y = \frac{2x+1}{x+1} \to R - \left[\frac{2}{1}\right] \to R - [2]$

ح) $y = \sqrt{\frac{2x+4}{x+1}} \to R - \left[\frac{2}{1}\right] \to [0, +\infty) - [\sqrt{2}]$

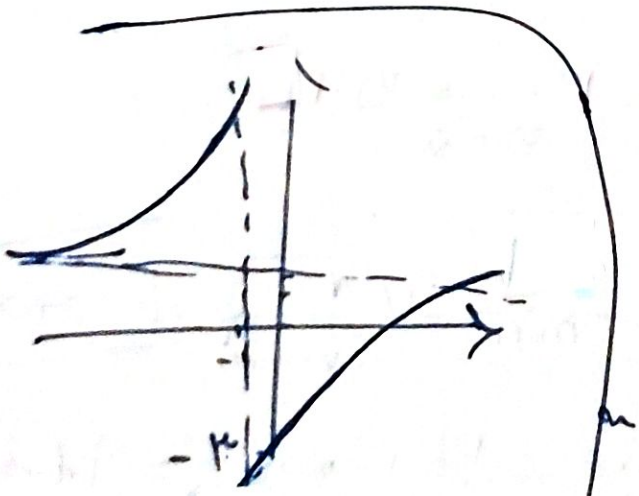
ط) $y = \sqrt{\frac{2x+1}{2-x}} \to R - [-2]$ متقاربين $\to [0, +\infty)$

ي) $y = \frac{2x-2}{x+1}$

$x + 1 = 0 \to x = -1$

$\frac{a}{c} = 2$

$x = 0 \to y = -\frac{2}{1} = -2$

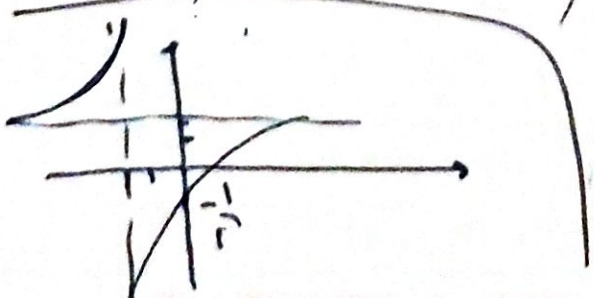


ك) $y = \frac{2x-1}{x+2}$

$x + 2 = 0 \to x = -2$

$\frac{a}{c} = 2$

$x = 0 \to y = -\frac{1}{2}$



۷) د

الف) $\sin x + \frac{1}{\sin x} \to R_f = (-\infty, -2] \cup [2, +\infty)$

ب) $\frac{x^6+1}{x^3} \to x^3 + \frac{1}{x^3}$ $\quad R_f = (-\infty, -2] \cup [2, +\infty)$

ج) $y = \frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x}} = \frac{\sqrt[3]{x^2}}{\sqrt[3]{x}} + \frac{1}{\sqrt[3]{x}} \to \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$

$R_f = (-\infty, -2] \cup [2, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \to R_f [2, +\infty)$

۸) د

الف) $x^2 + \frac{1}{x^2+3} + 3 - 3 \to \left\{ [\frac{10}{3}, +\infty) - 3 \to R_f = [\frac{1}{3}, +\infty) \right.$

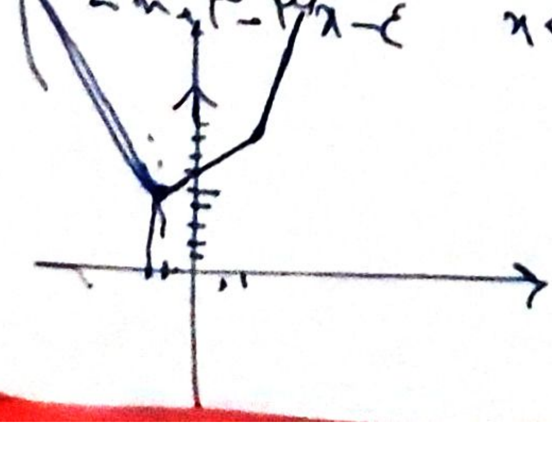
چون نمی‌تواند ۲ باشد پس

ب) $\sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}} \to R_f = [\frac{5}{2}, +\infty)$

$\sqrt{x^2+4} \leftarrow$ جدا نمی‌تواند ۱ شود

۹) د

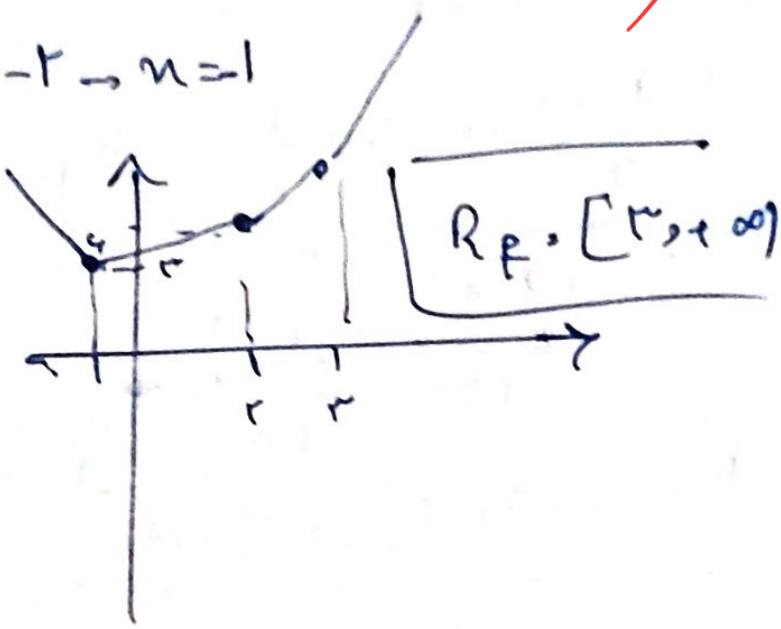
$$\begin{cases} x-3+3x+4 & x \geq 3 \\ -x+3+3x+4 & -\frac{4}{3} < x < 3 \\ -x+3-3x-4 & x < -\frac{4}{3} \end{cases} \quad \begin{cases} 4x+1 & x \geq 3 \\ 2x+7 & -\frac{4}{3} < x < 3 \\ -4x-1 & x \leq -\frac{4}{3} \end{cases}$$



$y = |x-2| + |2x+2|$

$x-2=0 \quad x=2 \qquad 2x=-2 \to x=-1$

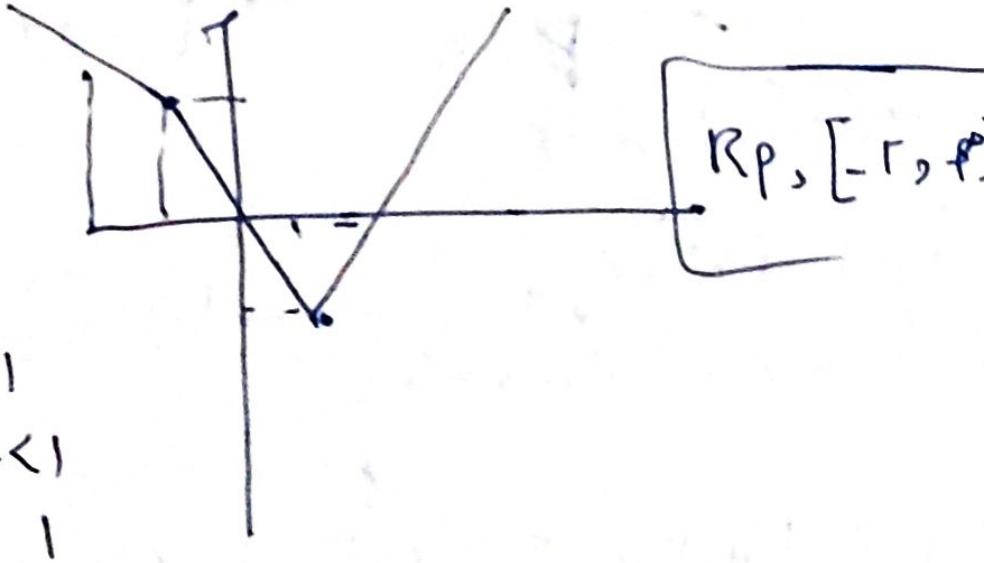
x	$-\infty$		-1		2		$+\infty$
$x-2$		$-$		$-$	0	$+$	
$2x+2$		$-$	0	$+$		$+$	
		$+$		$-$		$+$	



$$f(x) = \begin{cases} -2x & x \leq -1 \\ x+2 & -1 < x < 2 \\ 2x & x \geq 2 \end{cases}$$

$y = |2x-2| - |x+1|$

x	$-\infty$		-1		1		$+\infty$
$2x-2$		$-$		$-$	0	$+$	
$x+1$		$-$	0	$+$		$+$	
		$+$		$-$		$+$	



$$f(x) = \begin{cases} -x+2 & x \leq -1 \\ -2x+1 & -1 < x < 1 \\ x-2 & x \geq 1 \end{cases}$$