

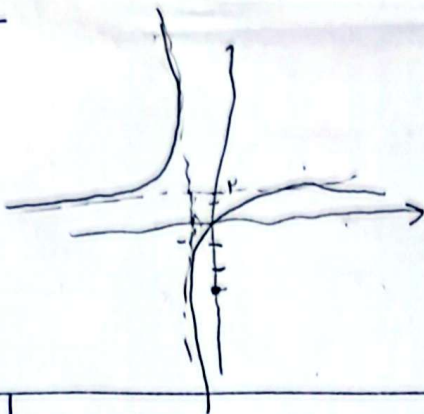
الف) $y = x^3 - 3x^2 + 3x$ $y = (x-1)^3 + 1 \rightarrow (y-1)^3 = x-1$ $\rightarrow x = (y-1)^3 + 1 \quad R = \mathbb{R}$	ب) $\frac{1}{x^2-2x} = y$ $yx^2 - 2yx = 1 \rightarrow yx^2 - 2yx - 1 = 0$ $\Delta = 4y^2 - 4 \times y \times (-1) = 4y^2 + 4y \geq 0$ $\hookrightarrow b^2 - 4ac \rightarrow 4y(y+1) \geq 0$ $R = (-\infty, -1] \cup [0, +\infty)$	۱
الف) $y = x^2 - 4x + 1$  $-\frac{b}{2a} = \frac{4}{2} = 2$ $4 - 16 + 1 = -11$ $R = [4, +\infty)$	ب) $y = -x^2 + 4x + 3$  $-\frac{b}{2a} = \frac{-4}{-2} = 2$ $-9 + 16 + 3 = 10$ $R = (-\infty, 10]$	۳
ج) $y = \sqrt{x^2 - 4x - 3}$ $\sqrt{(-1, +\infty)} \rightarrow R = [0, +\infty)$ $-\frac{b}{2a} = \frac{4}{2} = 2$ $4 - 16 - 3 = -15$	د) $y = \sqrt{4x - x^2}$ $\sqrt{(-\infty, 4]} \rightarrow R = [0, 4]$ $-\frac{b}{2a} = \frac{-4}{-2} = 2$ $-9 + 16 = 7$	۲
د) $y = x^4 - 4x^2 + 2x + 1$ $R = \mathbb{R}$	د) $y = x^4 - 4x^2 + 2x + 1$ $R = \mathbb{R}$	۳
ز) $y = \sqrt{x^2 - 4x^2 + 4x + 1}$ $R = [0, +\infty)$	ز) $y = (x^2 - 4x^2 + 4x + 1)$ $R = [0, +\infty)$	۳
ح) $y = \frac{2x+1}{x-2} = y$ $x = \frac{-y-1}{y-2} \quad R = \mathbb{R} - \{2\}$	ح) $y = \frac{2x+1}{x+1} = y$ $x = \frac{y-1}{y-2} \quad R = \mathbb{R} - \{2\}$	۴
ط) $y = \sqrt{\frac{2x+1}{x+1}}$ $y = \frac{2x+1}{x+1} \rightarrow yx + y = 2x + 1$ $\rightarrow yx - 2x = 1 - y \rightarrow x(y-2) = 1-y \rightarrow x = \frac{1-y}{y-2}$ $y = \sqrt{\mathbb{R} - \{2\}} \Rightarrow R = [0, +\infty) - \{2\}$ $R = \mathbb{R} - (2, 2)$	ط) $y = \sqrt{\frac{2x+1}{x-2}} = y = \frac{2x+1}{x-2}$ $\rightarrow x = \frac{-y+1}{y-2} \rightarrow y = \sqrt{\mathbb{R} - \{2\}}$ $R = [0, +\infty) - \{2\} \Rightarrow R = [0, +\infty)$ $R = [0, +\infty)$	۵

$$الف) y = \frac{x-3}{x+1}$$

جانب قائم = -1

جانب افقی = 3

$$x \rightarrow \infty \rightarrow y = -3$$

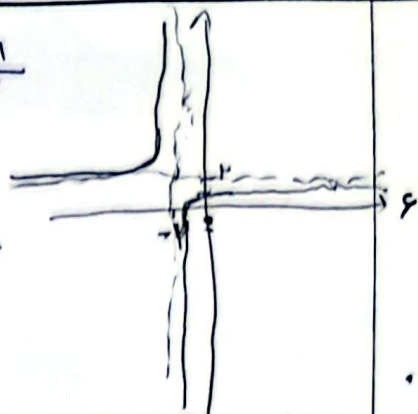


$$ب) y = \frac{x-1}{x+2}$$

جانب قائم = 2

جانب افقی = 1

$$x \rightarrow \infty \rightarrow y = -\frac{1}{2}$$



$$الف) y = \sinh x + \frac{1}{\sinh x}$$

$$R = (-\infty, -2] \cup [2, +\infty)$$

$$ب) y = \frac{x^2+1}{x^3} \rightarrow y = x^{-1} + \frac{1}{x^3}$$

$$R = (-\infty, -2] \cup [2, +\infty)$$

$$ج) y = \frac{\sqrt{x^2+1}}{\sqrt{x}} \rightarrow y = \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$R = (-\infty, -2] \cup [2, +\infty)$$

$$د) y = \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$R = [2, +\infty)$$

$$الف) y = x^2 + \frac{1}{x^2+3}$$

$$x^2+3-3 + \frac{1}{x^2+3} \rightarrow (x^2+3) + \frac{1}{(x^2+3)} - 3$$

$$\text{میزان} \rightarrow 3 + \frac{1}{3} - 3 = \frac{1}{3}$$

$$R = [\frac{1}{3}, +\infty)$$

$$ب) y = \frac{x^2+d}{\sqrt{x^2+e}}$$

$$\frac{x^2+e+1}{\sqrt{x^2+e}} = \sqrt{x^2+e} + \frac{1}{\sqrt{x^2+e}}$$

$$R = [\frac{d}{e}, +\infty)$$

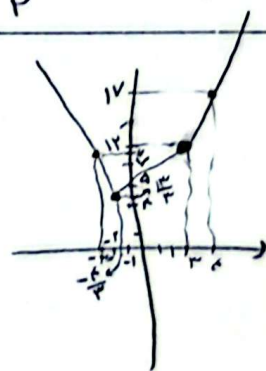
$$x \rightarrow \infty \rightarrow 3 + \frac{1}{3} = \frac{d}{e}$$

$$y = |x-3| + |3x+6| \rightarrow x = -\frac{6}{3}$$

$$x = 3$$

$$\begin{cases} x-3+3x+6 & x > 3 \\ -x+3+3x+6 & -\frac{6}{3} < x < 3 \\ -x+3-3x-6 & x < -\frac{6}{3} \end{cases}$$

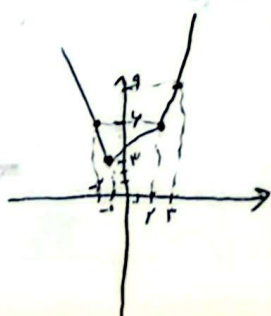
$$\begin{cases} 4x+3 & x > 3 \\ 2x+9 & -\frac{6}{3} < x < 3 \\ -4x-3 & x < -\frac{6}{3} \end{cases}$$



$$R = [\frac{13}{3}, +\infty)$$

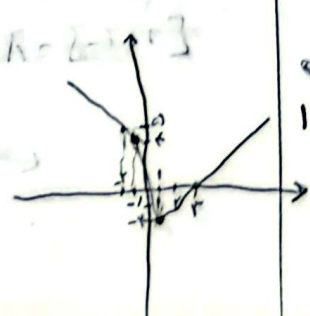
$$الف) y = |x-1| + |2x+2| \rightarrow x = -1 \quad R = [3, +\infty)$$

$$\begin{cases} -x-3 & x < -1 \\ x+2 & -1 < x < 1 \\ -x-1 & x > 1 \end{cases}$$



$$ب) y = |2x-2| - |x+1| \rightarrow x = -1$$

$$\begin{cases} -x+3 & x < -1 \\ -2x+1 & -1 < x < 1 \\ x-2 & x > 1 \end{cases}$$



$$R = [-3, +\infty)$$