

الف) $y = x^3 - 3x^2 + 3x$

$y = (x-1)^3 + 1 \rightarrow (y-1)^3 = x-1$

$\rightarrow x = (y-1)^3 + 1 \quad R = \mathbb{R}$

ب) $\frac{1}{x^2-2x} = y$

$yx^2 - 2yx = 1 \rightarrow yx^2 - 2yx - 1 = 0$

$\Delta = 4y^2 - 4 \times y \times (-1) = 4y^2 + 4y \geq 0$
 $\hookrightarrow y^2 + y \geq 0$

$\rightarrow y(y+1) \geq 0$

$R = (-\infty, -1] \cup [0, +\infty)$

الف) $y = x^2 - 4x + 1$



$-\frac{b}{2a} = \frac{4}{2} = 2$
 $f - 4 + 1 = -3$

$R = [4, +\infty)$

ب) $y = -x^2 + 4x + 3$



$-\frac{b}{2a} = \frac{-4}{-2} = 2$
 $-9 + 4 + 3 = -2$

$R = (-\infty, 12]$

ج) $y = \sqrt{x^2 - 4x - 3}$

$\sqrt{(-1, +\infty)} \rightarrow R = [0, +\infty)$

$-\frac{b}{2a} = \frac{4}{2} = 2$
 $f - 4 - 3 = -7$

د) $y = \sqrt{4x - x^2}$

$\sqrt{(-\infty, 4]} \rightarrow R = [0, 4]$

$-\frac{b}{2a} = \frac{-4}{-2} = 2$
 $-9 + 4 = -5$

د) $y = x^4 - 4x^2 + 2x + 1$

$R = \mathbb{R}$

ه) $y = x^4 - 4x^2 + 2x + 1$

$R = \mathbb{R}$

ز) $y = \sqrt{x^2 - 4x^2 + 4x + 1}$

$R = [0, +\infty)$

ز) $y = (x^2 - 4x^2 + 4x + 1)^2$

$R = [0, +\infty)$

ح) $y = \frac{2x+1}{x-2} = y$

$x = \frac{-2y-1}{y-2} \quad R = \mathbb{R} - \{2\}$

ح) $y = \frac{2x+1}{x-2} = y$

$x = \frac{y-1}{y-2} \quad R = \mathbb{R} - \{2\}$

ط) $y = \sqrt{\frac{2x+1}{x-2}}$: $y = \frac{2x+1}{x-2} \rightarrow yx - y = 2x + 1$

$\rightarrow yx - 2x = 1 - y \rightarrow x(y-2) = 1-y \rightarrow x = \frac{1-y}{y-2}$

$y = \sqrt{\mathbb{R} - \{2\}} \Rightarrow R = [0, +\infty) - \{5/2\}$
 $R = \mathbb{R} - (2, 5/2)$

ط) $y = \sqrt{\frac{2x+1}{x-2}} = y = \frac{2x+1}{x-2}$

$\rightarrow x = \frac{-2y+1}{y-2} \rightarrow y = \sqrt{\mathbb{R} - \{2\}}$

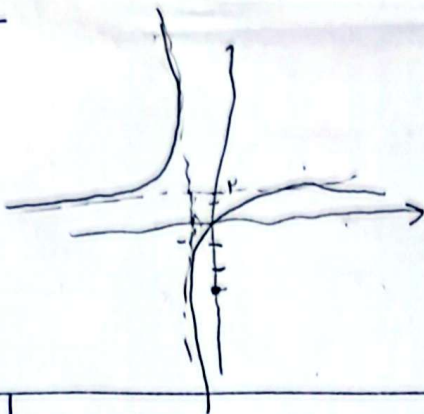
$R = [0, +\infty) - \{5/2\} \Rightarrow R = [0, +\infty)$

$$الف) y = \frac{x-3}{x+1}$$

جانب قاع = -1

جانب اعلى = 3

$$x \rightarrow y = -3$$

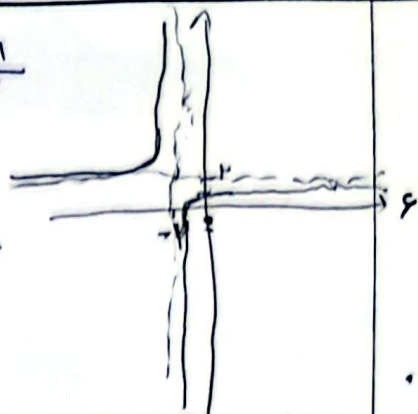


$$ب) y = \frac{x-1}{x+2}$$

جانب قاع = 2

جانب اعلى = 1

$$x \rightarrow y = -\frac{1}{2}$$



$$الذ) y = \sinh x + \frac{1}{\sinh x}$$

$$R = (-\infty, -2] \cup [2, +\infty)$$

$$ب) y = \frac{x^2+1}{x^3} \rightarrow y = x^{-1} + \frac{1}{x^3}$$

$$R = (-\infty, -2] \cup [2, +\infty)$$

$$ج) y = \frac{\sqrt{x^2+1}}{\sqrt{x}} \rightarrow y = \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$R = (-\infty, -2] \cup [2, +\infty)$$

$$د) y = \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$R = [2, +\infty)$$

$$الذ) y = x^2 + \frac{1}{x^2+3}$$

$$x^2+3-3 + \frac{1}{x^2+3} \rightarrow (x^2+3) + \frac{1}{(x^2+3)} - 3$$

$$مميزه \rightarrow 3 + \frac{1}{x} - 3 = \frac{1}{x}$$

$$R = [\frac{1}{x}, +\infty)$$

$$ب) y = \frac{x^2+d}{\sqrt{x^2+e}}$$

$$\frac{x^2+e+1}{\sqrt{x^2+e}} = \sqrt{x^2+e} + \frac{1}{\sqrt{x^2+e}}$$

$$R = [\frac{d}{e}, +\infty)$$

$$x \rightarrow 2 + \frac{1}{x} = \frac{d}{e}$$

$$y = |x-3| + |3x+6| \rightarrow x = -\frac{6}{3}$$

$$x = 3$$

$$\begin{cases} x-3+3x+6 & x > 3 \\ -x+3+3x+6 & -\frac{6}{3} < x < 3 \\ -x+3-3x-6 & x < -\frac{6}{3} \end{cases}$$

$$x > 3$$

$$-\frac{6}{3} < x < 3$$

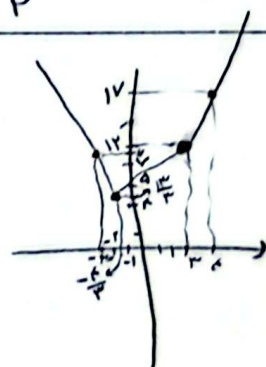
$$x < -\frac{6}{3}$$

$$\begin{cases} x-3 & x > 3 \\ 3x+6 & -\frac{6}{3} < x < 3 \\ -x+3 & x < -\frac{6}{3} \end{cases}$$

$$x > 3$$

$$-\frac{6}{3} < x < 3$$

$$x < -\frac{6}{3}$$



$$R = [\frac{13}{3}, +\infty)$$

$$الذ) y = |x-1| + |2x+2|$$

$$x = 1$$

$$x = -1$$

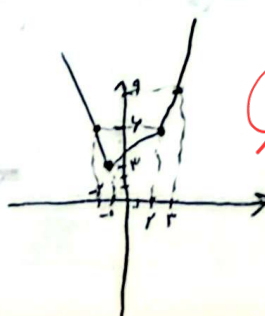
$$R = [3, +\infty)$$

$$\begin{cases} -x-1 & x < -1 \\ x+2 & -1 < x < 1 \\ -x & x > 1 \end{cases}$$

$$x < -1$$

$$-1 < x < 1$$

$$x > 1$$



$$ب) y = |2x-1| - |x+1|$$

$$x = \frac{1}{2}$$

$$x = -1$$

$$R = [-1, +\infty)$$

$$\begin{cases} -x+1 & x < -1 \\ -2x+1 & -1 < x < \frac{1}{2} \\ x-1 & x > \frac{1}{2} \end{cases}$$

$$R = [-1, +\infty)$$

