14,50

هلیا عقاری

الف) $x^3 - 3x^2 + 3x + 1 = y - 1 \Rightarrow R_f = R$ ①

$(x-1)^3 = y-1 \Rightarrow x-1 = \sqrt[3]{y-1} \Rightarrow x = \sqrt[3]{y-1} + 1$

ب) $y = \frac{1}{x^2 - 2x} \Rightarrow yx^2 - 2xy = 1$ ②

$yx^2 - 2yx - 1 = 0 \quad \Delta \to 4y^2 - (4)(-1)(y) = 4y^2 + 4y$

$x = \frac{2y \pm \sqrt{4y^2+4y}}{2y} \quad 4y^2 + 4y \geq 0 \quad 4y(y+1)$

$2y \neq 0 \to y \neq 0$

1	0	
+	-	+

$(-\infty, -1] \cup (0, +\infty)$

الف) $x^2 - 4x + 10$ ③

$\Delta = 16 - 40 = -24$

$-\frac{b}{2a} = \frac{4}{2} = 2$

$\frac{-\Delta}{4a} = \frac{24}{4} = 6$

10/1

ضریب $x^2 > 0$ $\Rightarrow R_f = [6, +\infty)$

عرض کمی $= \frac{-\Delta}{4a} = \frac{-(36+12)}{-4} = 11$

ب) $-x^2 + 6x + 3$

ضریب < 0

$\longrightarrow R_f: (-\infty, 12]$

عرض کمی $= \frac{-\Delta}{4a} = \frac{-(16+12)}{4} = -7$

ج) $\sqrt{x^2 - 4x - 4}$

$R_f = \sqrt{[-8, +\infty)} \to R_f = [0, +\infty)$

② د) $\sqrt{-x^2+4x}$ عرض اکسترمم $= \frac{-\Delta}{4a} = \frac{-(36)}{-4} = 9$

$R_f\ \sqrt{(-\infty, 9]} \Rightarrow R_f = [0, \cancel{9}\ 3]$

③ د)

الف) $y = x^3 - 5x^2 + 2x + 1$ توان‌ها همه فرد $\rightarrow$ $R_f = R$

ب) $y = x^7 - 4x^2 + 3x + 2$ نکته بالا $\rightarrow R_f = R$

ج) $\sqrt{x^7 - 4x^2 + 5x + 1} \Rightarrow \sqrt{R} = [0, +\infty) = R_f$

د) $(\underbrace{x^5 - 4x^2 + 3x + 1}_{R})^2 \rightarrow R_f = [0, +\infty)$

④ د)

الف) $y = \frac{3x+1}{x-3}$ $R_f = R - \{3\}$

ب) $y = \frac{2x+1}{x+1}$ $R_f = R - \{2\}$

⑤ د)

الف) $\sqrt{\frac{2x+4}{x+1}} = \sqrt{R - \{2\}} = [0, +\infty) - \{\sqrt{2}\}$

ب) $\sqrt{\frac{3x+1}{-x+2}} = \sqrt{R - \{-3\}} = [0, +\infty)$

⑥

الف) $2x - 3$ ...

الف) $\frac{2x-3}{x+1}$ مجانب قائم $x=-1$

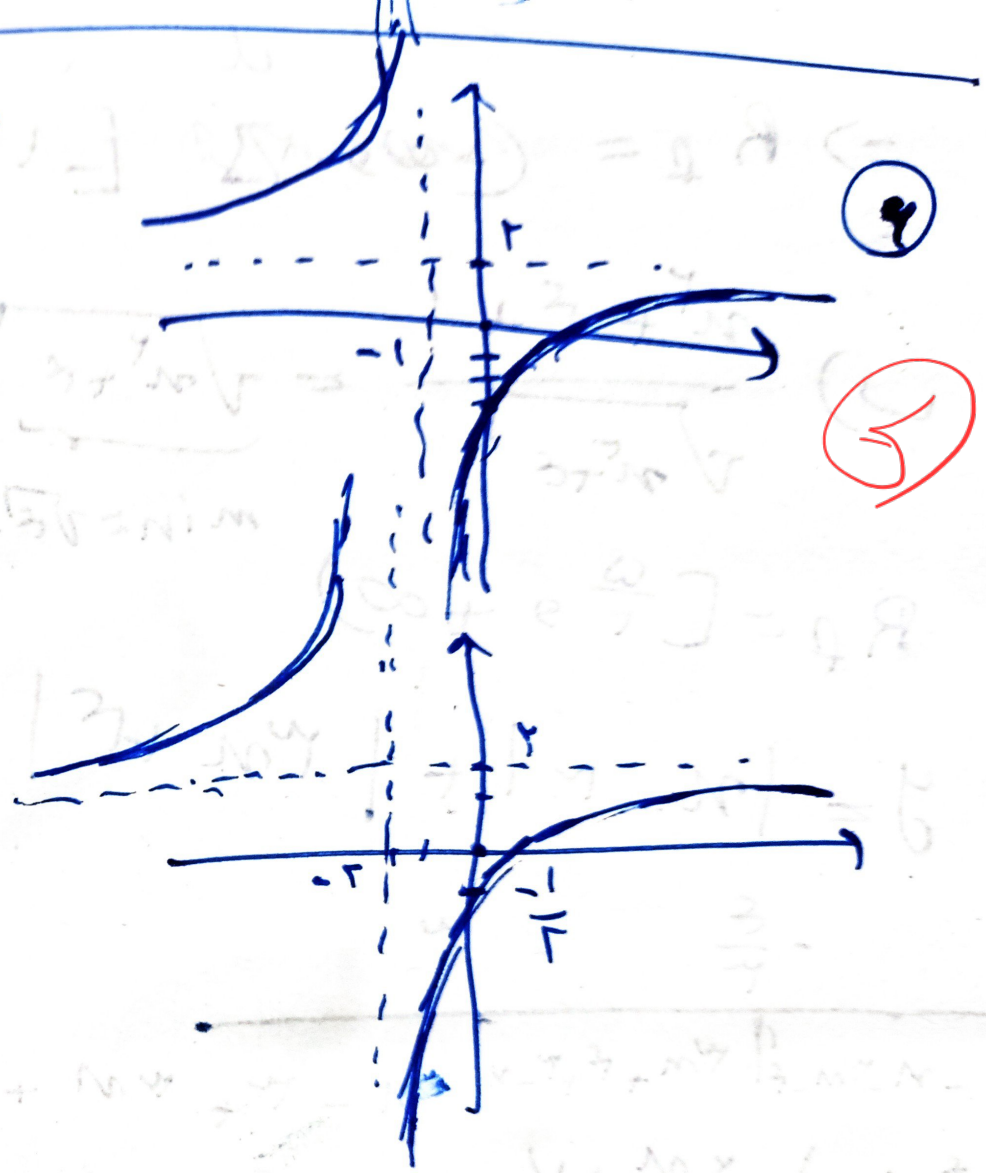
مجانب افقی $y=2$

$x=0 \rightarrow -3$

ب) $\frac{2x-1}{x+2}$ مجانب قائم $x=-2$

مجانب افقی $y=2$

$x=0 \rightarrow y=\frac{-1}{2}$



ب) $y = \sin x + \frac{1}{\sin x}$ $R_f = (-\infty, -2] \cup [2, +\infty)$ (7)

ب) $y = \frac{x^4+1}{x^3} = x + \frac{1}{x^3}$ $R_f = (-\infty, -2] \cup [2, +\infty)$ (5)

ج) $y = \frac{\sqrt[3]{x^2+1}}{\sqrt[3]{x}} = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$ $R_f = (-\infty, -2] \cup [2, +\infty)$

ب) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \rightarrow R_f = [2, +\infty)$

الف) $x^2 + \frac{1}{x^2+2} \rightarrow x^2 + 2 + \frac{1}{x^2+2} - 2$ (8)

$x^2 + 2 = a$ عبارت مثبت $a > 0$ $\rightarrow a + \frac{1}{a} \rightarrow a + \frac{1}{a} \geq 2 \rightarrow a + \frac{1}{a} - 2 \geq -1$

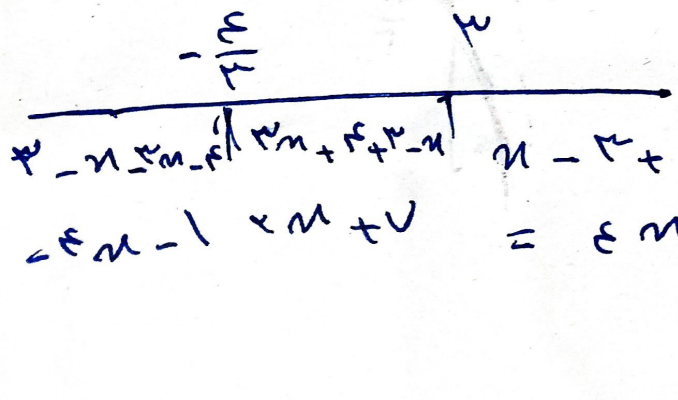
$\Rightarrow R_f = [-1, +\infty)$ (1,5)

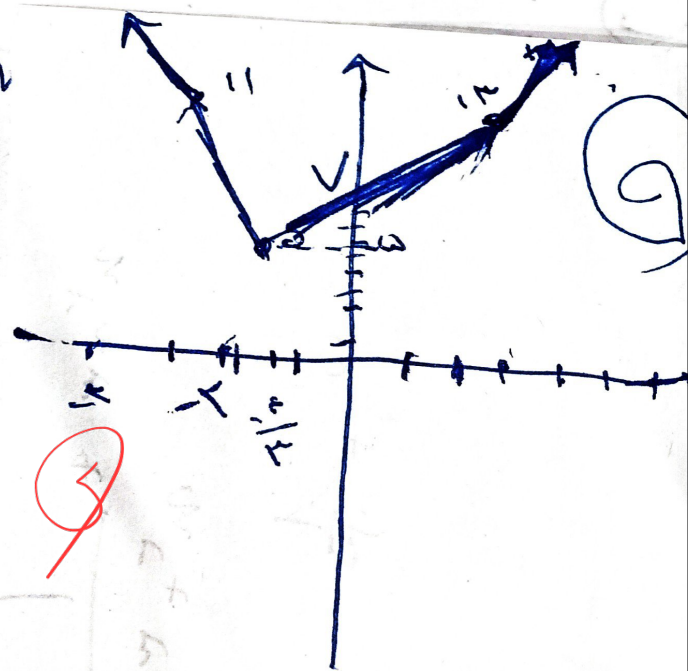
ب) $\frac{x^2+4+1}{\sqrt{x^2+4}} = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}} \Rightarrow \min = 2 + \frac{1}{2} = \frac{5}{2}$

$\min = \sqrt{4} = 2$

$R_f = [\frac{5}{2}, +\infty)$

$y = |x-2| + |3x+2|$





(9)

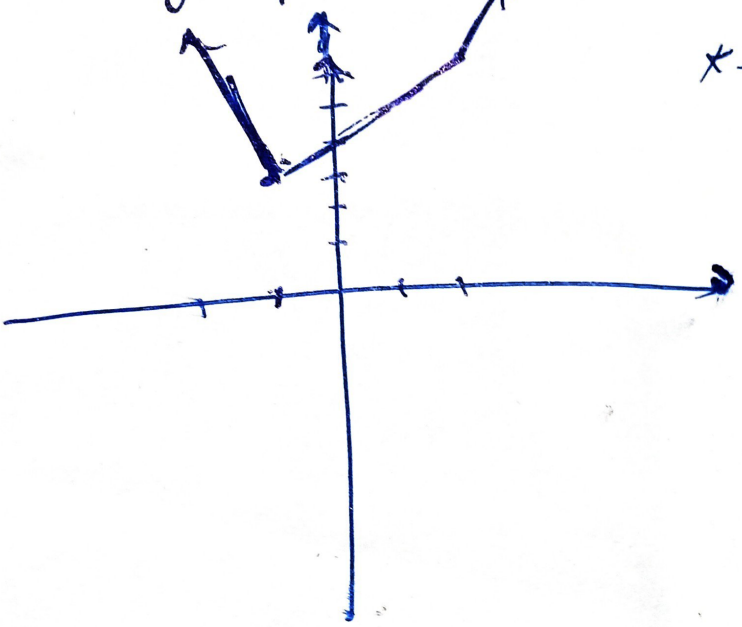
$x-2+3x+2 = 4x$

$-x+2-3x-2 = -4x$

(9)

(10)

ا) $y = |x-2| + |2x+2|$

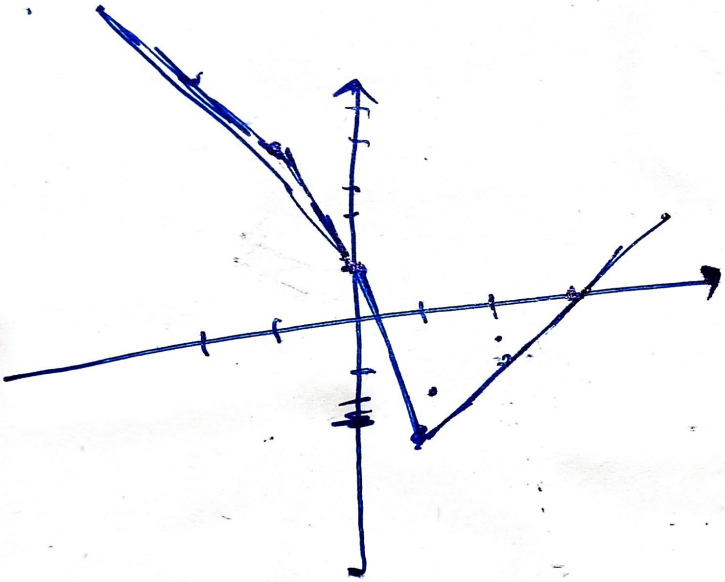


-1	2
$-x+2-2x-2$	$x-2+2x+2$
$-3x$	$3x$

$R_f = [3, +\infty)$

5

ب) $|2x-2| - |x+1|$



-1	1
$2-2x+x+1$	$1-x-2-x$
$-x+3$	$3-x$

$R_f = [-2, +\infty)$

ج) $x^2+2+\frac{1}{x^2+2}-2 \xrightarrow{x^2=u} u+\frac{1}{u}-2$ $\quad R=[\frac{1}{2}, +\infty)$

(ج) ١٧