

نام و نام خانوادگی: کلاس: شماره: تکلیف شماره: پاسخنامه تشریحی:
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الف) $y = x^3 - 3x^2 + 3x = x^3 - 3x^2 + 3x - 1 + 1 = y \Rightarrow (x-1)^3 = y-1 \Rightarrow x-1 = \sqrt[3]{y-1}$

ب) $y = \frac{1}{x^2-2x} \Rightarrow yx^2-2xy-1=0 \Rightarrow \Delta = 4y^2+4y = 4y(y+1) \geq 0 \Rightarrow y(y+1) \geq 0$
 $\Rightarrow x = \sqrt[3]{y-1} + 1$
 $R_f = \mathbb{R}$

$R_f = (-\infty, -1] \cup (0, +\infty)$

الف) $y = x^2 - 4x + 1 \Rightarrow \begin{cases} x_{min} = \frac{-b}{2a} = 2 \\ y_{min} = -3 \end{cases} \Rightarrow R_f = [-3, +\infty)$

ب) $y = -x^2 + 4x + 3 \Rightarrow \begin{cases} x_{min} = 0 \\ y_{max} = 7 \end{cases} \Rightarrow R_f = (-\infty, 7]$

ج) $y = \sqrt{x^2 - 4x - 5} \Rightarrow \begin{cases} x_{min} = 5 \\ y_{min} = -1 \end{cases} \Rightarrow R_f = [0, +\infty)$

د) $y = \sqrt{4x - x^2} \Rightarrow \begin{cases} x_{max} = 2 \\ y_{max} = 2 \end{cases} \Rightarrow R_f = [0, 2]$

الف) $y = x^3 - 6x^2 + 9x + 1 \Rightarrow R_f = \mathbb{R}$

ب) $y = x^3 - 4x^2 + 3x + 1 \Rightarrow R_f = \mathbb{R}$

ج) $y = \sqrt{x^3 - 4x^2 + 3x + 1} \Rightarrow R_f = [0, +\infty)$

د) $y = (x^3 - 6x^2 + 9x + 1)^2 \Rightarrow R_f = [0, +\infty)$

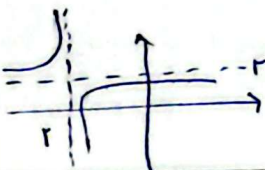
الف) $y = \frac{x+1}{x-1} \Rightarrow R_f = \mathbb{R} - \{1\}$

ب) $y = \frac{x+1}{x+1} = 1 \Rightarrow R_f = \{1\}$

الف) $y = \sqrt{\frac{x+1}{x+1}} \Rightarrow R_f = [0, +\infty) - \{1\}$

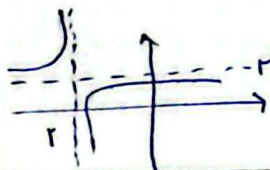
ب) $y = \sqrt{\frac{x+1}{x-1}} \Rightarrow R_f = [0, +\infty) - \{1\}$

$$y = \frac{2x-1}{x+1} \quad \begin{cases} \text{if } x \geq -1 \\ \text{if } x < -1 \end{cases}$$



5

$$y = \frac{x-1}{x+1} \quad \begin{cases} \text{if } x \geq -1 \\ \text{if } x < -1 \end{cases}$$



الف) $y = \sin x + \frac{1}{\sin x} \Rightarrow \begin{cases} \text{if } a > 0 \Rightarrow \sin x + \frac{1}{\sin x} \geq 2 \\ \text{if } a < 0 \Rightarrow \sin x + \frac{1}{\sin x} \leq -2 \end{cases} \quad R_f = (-\infty, -2] \cup [2, +\infty)$

ب) $y = \frac{x^2+1}{x^2} \Rightarrow x^2 + \frac{1}{x^2} \Rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

ج) $y = \frac{\sqrt{x^2+1}}{\sqrt{x}} \Rightarrow \sqrt{x} + \frac{1}{\sqrt{x}} \Rightarrow (-\infty, -1] \cup [1, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \Rightarrow [1, +\infty)$

5

الف) $y = x^2 + \frac{1}{x^2+1} \Rightarrow y = \frac{x^2+x^2+1}{x^2+1} \Rightarrow \frac{x^2(x^2+1)}{x^2+1} \Rightarrow x^2 \Rightarrow [0, +\infty)$

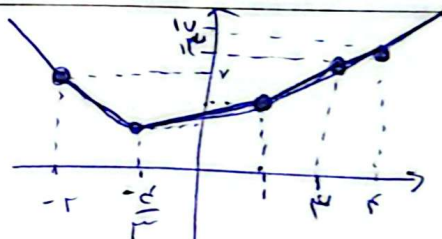
ب) $y = \frac{x^2+1}{\sqrt{x^2+1}} \Rightarrow \frac{x^2+1}{\sqrt{x^2+1}} \Rightarrow \frac{x^2+1}{\sqrt{x^2+1}} + \frac{1}{\sqrt{x^2+1}} \Rightarrow [1, +\infty)$

$\sqrt{x^2+1} + \frac{1}{\sqrt{x^2+1}} \xrightarrow{x \rightarrow \infty} \sqrt{x} + \frac{1}{\sqrt{x}} \Rightarrow [\frac{1}{\sqrt{x}}, +\infty)$

10

$$y = |x-1| + |x+1|$$

$$\begin{cases} x-1 + x+1 = 2x & x \geq 1 \\ 1-x + x+1 = 2 & -1 \leq x < 1 \\ -x+1 + x+1 = 2 & x < -1 \end{cases}$$

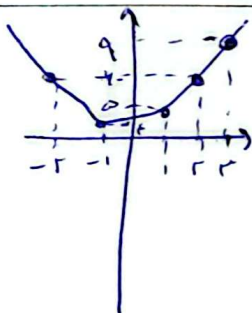


1, 0

الف) $y = |x-1| + |x+1|$

$$\begin{cases} -x+1 - x-1 = -2x & x < -1 \\ -x+1 + x+1 = 2 & -1 \leq x < 1 \\ x-1 + x+1 = 2x & x \geq 1 \end{cases}$$

$R = [1, +\infty)$



$y = |x-1| - |x+1| = \begin{cases} x-1 - x-1 = -2 & x < -1 \\ x-1 - x+1 = 0 & -1 \leq x < 1 \\ x-1 + x+1 = 2x & x \geq 1 \end{cases}$

$R = [-1, +\infty)$