Year: Month: Date: ( ) «ریاضی امانی» Subject: تکلیف شماره ۶

۱. برد به روش اصلی

$\Delta = 9 - 12 = -3 \Rightarrow \Delta < 0$ ریشه ندارد.

الف) $y = x^3 - 3x^2 + 3x \Rightarrow x(x^2 - 3x + 3)$ $R_f = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x + 1 - 1}$

$y = \frac{1}{(x-1)^2 - 1}$ $y(x-1)^2 - 1 = 1 \Rightarrow y(x-1)^2 = 2$ $(x-1)^2 = \frac{2}{y} \Rightarrow x - 1 = \sqrt{\frac{2}{y}}$

$\Rightarrow x = \sqrt{\frac{2}{y}} + 1 \Rightarrow \frac{2}{y} \geq 0 \Rightarrow y \geq 2$

۲. الف) $y = x^2 - 4x + 10 \xrightarrow{a>0} -\frac{\Delta}{4a} = \frac{-16 - 40}{4} = 6$ $R_f = [6, +\infty)$

ب) $y = -x^2 + 6x + 3 \xrightarrow{a<0} -\frac{\Delta}{4a} = \frac{-36 + 12}{-4} = 12$ $R_f = (-\infty, 12]$

ج) $y = \sqrt{x^2 - 4x - 3} \xrightarrow{a>0} -\frac{\Delta}{4a} = \frac{-16 + 12}{4} = -7$ $R_f = \sqrt{[-7, +\infty)} = [0, +\infty)$

د) $y = \sqrt{6x - x^2} \xrightarrow{a<0} -\frac{\Delta}{4a} = -\frac{36}{-4} = 9$ $R_f = \sqrt{(-\infty, 9]} = [0, 3]$

۳. الف) $y = x^3 - 5x^2 + 2x + 1$ $R_f = \mathbb{R}$ <= توان فرد

ب) $y = x^7 - 6x^4 + 3x + 2$ $R_f = \mathbb{R}$

ج) $y = \sqrt{x^7 - 6x^2 + 5x + 1}$ $R_f = \sqrt{\mathbb{R}} = [0, +\infty)$

د) $y = (x^5 - 4x^2 + 3x + 1)^2$ $R_f = \mathbb{R}$

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۴- الف) $y=\frac{۳x+۱}{x-۲}$ $R_f = \mathbb{R}-\{\frac{a}{c}\} = \mathbb{R}-\{۳\}$

ب) $y=\frac{۲x+۱}{x+۱}$ $R_f = \mathbb{R}-\{۲\}$

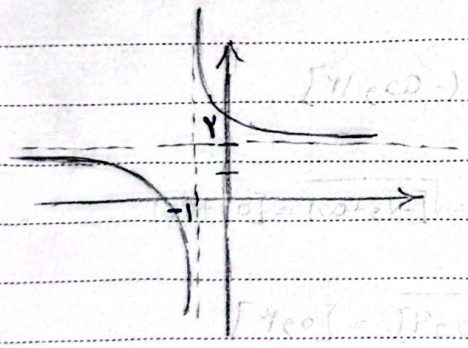
۵- الف) $y=\sqrt{\frac{۲x+۴}{x+۱}}$ $R_f = \sqrt{\mathbb{R}-\{۲\}} = [۰,+\infty)-\{\sqrt{۲}\}$

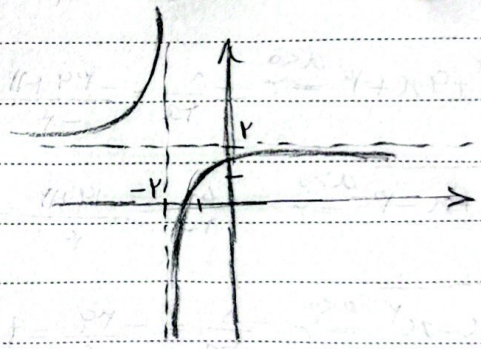
ب) $y=\sqrt{\frac{۳x+۱}{۲-x}}$ $R_f = \sqrt{\mathbb{R}-\{-۳\}} = [۰,+\infty)$

۶- نمودار توابع رسم شود.

الف) $\frac{۲x-۳}{x+۱}$ ب) $\frac{۲x-۱}{x+۲}$

$x=-۱$ = ریشه مخرج $y=\frac{a}{c}=۲$ افقی -۲ = ریشه مخرج $=x$ $y=\frac{a}{c}=۲$





۷- الف) $y=\sin x+\frac{۱}{\sin x}$ $\sin x \neq ۰$

if $\sin x>۰ \Rightarrow \sin x+\frac{۱}{\sin x}\geq ۲ \Rightarrow [۲,+\infty)$
if $\sin x<۰ \Rightarrow \sin x+\frac{۱}{\sin x}\leq ۲ \Rightarrow (-\infty,۲]$
$\Big\}$ $R_f=(-\infty,۲]\cup[۲,+\infty)$

ب) $\frac{x^۴+۱}{x^۲} = x^۲+\frac{۱}{x^۲}$ $a>۰ \Rightarrow [۲,+\infty)$ $a<۰ \Rightarrow (-\infty,۲]$ $\Rightarrow R_f=(-\infty,۲]\cup[۲,+\infty)$

ج) $\frac{\sqrt[۳]{x^۲}+۱}{\sqrt[۳]{x}} = \sqrt[۳]{x}+\frac{۱}{\sqrt[۳]{x}}$ $R_f=(-\infty,۲]\cup[۲,+\infty)$

د) $\sqrt{x}+\frac{۱}{\sqrt{x}}$ $a=\sqrt{x} \Rightarrow a>۰ \Rightarrow R_f=[۲,+\infty)$

$x < -\frac{3}{2}$ $\begin{cases} x-3<0 & -x+3 \\ 2x+3<0 & -2x-3 \end{cases}$ $\quad -\frac{3}{2} \le x < 3$ $\begin{cases} x-3<0 \Rightarrow -x+3 \\ 2x+3 \ge 0 \Rightarrow 2x+3 \end{cases}$ $\quad x \ge 3$ $\begin{cases} x-3 \ge 0 \Rightarrow x-3 \\ 2x+3 \ge 0 \Rightarrow 2x+3 \end{cases}$

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۷- الف) $y = x^2 + \frac{1}{x^2+2} \Rightarrow y = \frac{x^2(x^2+2)+1}{x^2+2} \Rightarrow y(x^2+2) = x^2(x^2+2)+1$

$\Rightarrow yx^2 + 2y = x^4 + 2x^2 + 1 \Rightarrow x^4 + 2x^2 + 1 - yx^2 - 2y = 0 \Rightarrow x^4 + x^2(2-y) + (1-2y) = 0$

$\Delta = (2-y)^2 - 4(1-2y) \ge 0 \Rightarrow y^2 - 4y + 4 - 4 + 8y \ge 0 \Rightarrow y^2 + 4y \ge 0 \Rightarrow (y+1)(y+5) \ge 0$

$y = -1, y = -5 \xrightarrow{x^2 \ge 0}$ $R_f = [\frac{1}{2}, +\infty)$

ب) $y = \frac{x^2+5}{\sqrt{x^2+4}}$ $\quad x^2+4 \ge 0 \Rightarrow x \in \mathbb{R}$ $\quad R_f = [\frac{5}{2}, +\infty)$

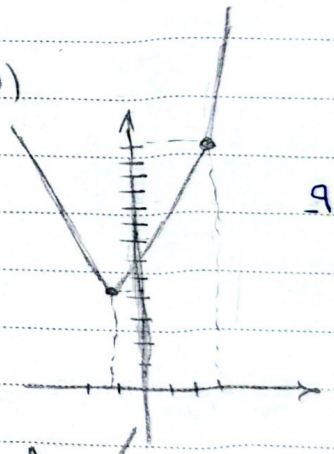
$x^2 \ge 0 \Rightarrow x^2+4 \ge 4 \Rightarrow \sqrt{x^2+4} \ge 2$

$\frac{x^2+5}{\sqrt{x^2+4}} = \frac{(x^2+4)+1}{\sqrt{x^2+4}} = \frac{x^2+4}{\sqrt{x^2+4}} + \frac{1}{\sqrt{x^2+4}} = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}}$ راه ۲

$a = \sqrt{x^2+4} \ge 2 \Rightarrow y = a + \frac{1}{a} = \frac{5}{2} = 2.5 \Rightarrow R_f = [2.5, +\infty)$

۹- $y = |x-3| + |2x+3|$ $\quad -\frac{3}{2}$

$y = \begin{cases} x-3+2x+3 & x > 3 \\ -x+3+2x+3 & -\frac{3}{2} \le x \le 3 \\ -x+3-2x-3 & x < -\frac{3}{2} \end{cases} \Rightarrow \begin{cases} 3x & x \ge 3 \\ x+6 & -\frac{3}{2} \le x < 3 \\ -3x & x < -\frac{3}{2} \end{cases}$



۱۰- الف) $y = |x^2-2| + |2x^{-1}+2|$

$\begin{vmatrix} 2 \\ 4 \end{vmatrix}$	$\begin{vmatrix} 3 \\ 9 \end{vmatrix}$	$\begin{vmatrix} -1 \\ 3 \end{vmatrix}$	$\begin{vmatrix} -2 \\ 6 \end{vmatrix}$

ب) $y = |2x^{+1} - 2| - |x^{-1} + 1|$

$\begin{vmatrix} +1 \\ -2 \end{vmatrix}$	$\begin{vmatrix} +2 \\ -1 \end{vmatrix}$	$\begin{vmatrix} -1 \\ 4 \end{vmatrix}$	$\begin{vmatrix} -2 \\ +5 \end{vmatrix}$