17

Year: Month: Date: ( ) Subject: تکلیف شماره ۶ "ریاضیات امانی"

۱. برد به روش اصلی

$\Delta = 9 - 12 = -3 \Rightarrow \Delta < 0$. ریشه ندارد

الف) $y = x^3 - 3x^2 + 3x \Rightarrow x(x^2 - 3x + 3)$ $\quad R_f = \mathbb{R}$ $\quad$ 1

$yx^2 - 2xy = 1 \rightarrow x = \dfrac{2y \pm \sqrt{4y^2 + 4y}}{2y} \Rightarrow 4y^2 + 4y \geq 0,\ y \neq 0$

ب) $y = \dfrac{1}{x^2 - 2x + 1 - 1}$ $\quad R = (-\infty, -1] \cup (0, +\infty)$

$y = \dfrac{1}{(x-1)^2 - 1}$ $\quad y(x-1)^2 - 1 = 1 \Rightarrow y(x-1)^2 = 2 \quad (x-1)^2 = \dfrac{2}{y} \Rightarrow x - 1 = \sqrt{\dfrac{2}{y}}$

$\Rightarrow x = \sqrt{\dfrac{2}{y}} + 1 \Rightarrow \dfrac{2}{y} \geq 0 \Rightarrow y \geq 2$

۲ الف) $y = x^2 - 4x + 10 \overset{a>0}{\Longrightarrow} -\dfrac{\Delta}{4a} = \dfrac{-(16-40)}{4} = 6$ $\quad R_f = [6, +\infty)$

ب) $y = -x^2 + 6x + 3 \overset{a<0}{\Longrightarrow} -\dfrac{\Delta}{4a} = \dfrac{-(36+12)}{-4} = 12$ $\quad R_f = (-\infty, 12]$ $\quad$ 5

ج) $y = \sqrt{x^2 - 4x - 3} \overset{a>0}{\Longrightarrow} -\dfrac{\Delta}{4a} = \dfrac{-(16+12)}{4} = -7$ $\quad R_f = \sqrt{[-7, +\infty)} = [0, +\infty)$

د) $y = \sqrt{6x - x^2} \overset{a<0}{\Longrightarrow} -\dfrac{\Delta}{4a} = \dfrac{-36}{-4} = 9$ $\quad R_f = \sqrt{(-\infty, 9]} = [0, 3]$

۳ الف) $y = x^3 - 5x^2 + 2x + 1$ $\quad R_f = \mathbb{R}$. توان فرد ⇐

ب) $y = x^7 - 6x^4 + 3x + 2$ $\quad R_f = \mathbb{R}$ $\quad$ 1,5

ج) $y = \sqrt{x^7 - 6x^2 + 5x + 1}$ $\quad R_f = \sqrt{\mathbb{R}} = [0, +\infty)$

د) $y = (x^5 - 4x^2 + 3x + 1)^2$ $\quad R_f = \mathbb{R}$ $[0, +\infty)$

Year: Month: Date: ( ) Subject:

۴- الف) $y=\frac{3x+1}{x-2}$ $R_f = \mathbb{R}-\{\frac{a}{c}\}=\mathbb{R}-\{3\}$ (۱)

ب) $y=\frac{2x+1}{x+1}$ $R_f=\mathbb{R}-\{2\}$

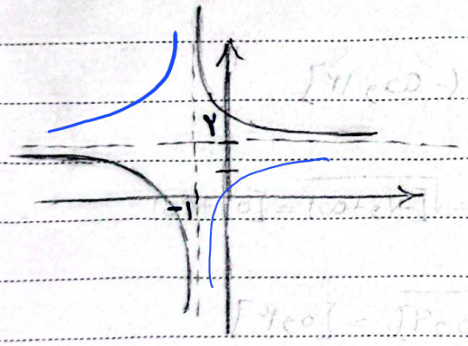
۵- الف) $y=\sqrt{\frac{2x+4}{x+1}}$ $R_f=\sqrt{\mathbb{R}-\{2\}}=[0,+\infty)-\{\sqrt{2}\}$ (۱)

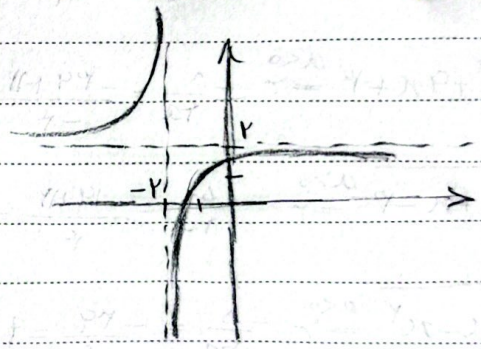
ب) $y=\sqrt{\frac{3x+1}{2-x}}$ $R_f=\sqrt{\mathbb{R}-\{-3\}}=[0,+\infty)$

۶- نمودار توابع رسم شود.

الف) $\frac{2x-3}{x+1}$ ب) $\frac{2x-1}{x+2}$ (۱,۵)

$x=-1$ = ریشه مخرج $y=\frac{a}{c}=2$ (عدد) -2 = ریشه مخرج $=x$ $y=\frac{a}{c}=2$





۷- الف) $y=\sin x+\frac{1}{\sin x}$ $\sin x\neq 0$ (۱)

if $\sin x>0 \Rightarrow \sin x+\frac{1}{\sin x}\geq 2 \Rightarrow [2,+\infty)$
if $\sin x<0 \Rightarrow \sin x+\frac{1}{\sin x}\leq -2 \Rightarrow (-\infty,-2]$
$\Rightarrow R_f=(-\infty,-2]\cup[2,+\infty)$

ب) $\frac{x^4+1}{x^2}=x^2+\frac{1}{x^2}$ $a>0 \Rightarrow [2,+\infty)$ $a<0 \Rightarrow (-\infty,-2]$ $\Rightarrow R_f=(-\infty,-2]\cup[2,+\infty)$

ج) $\frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x}}=\sqrt[3]{x}+\frac{1}{\sqrt[3]{x}}$ $R_f=(-\infty,-2]\cup[2,+\infty)$

د) $\sqrt{x}+\frac{1}{\sqrt{x}}$ $a=\sqrt{x} \Rightarrow a>0 \Rightarrow R_f=[2,+\infty)$

TOPCO

$x<-\frac{4}{3}\begin{cases} x-3<0 & -x+3 \\ 3x+4<0 & -3x-4 \end{cases}$ $-\frac{4}{3}\le x<3\begin{cases} x-3<0 \Rightarrow -x+3 \\ 3x+4\geq 0 \Rightarrow 3x+4 \end{cases}$ $x\geq 3\begin{cases} x-3\geq 0 \Rightarrow x-3 \\ 3x+4\geq 0 \Rightarrow 3x+4 \end{cases}$

۸- (۲)

الف) $y=x^2+\frac{1}{x^2+3} \Rightarrow y=\frac{x^2(x^2+3)+1}{x^2+3} \Rightarrow y(x^2+3)=x^2(x^2+3)+1$

$\Rightarrow yx^2+3y=x^4+3x^2+1 \Rightarrow x^4+3x^2+1-yx^2-3y=0 \Rightarrow x^4+x^2(3-y)+(1-3y)=0$

$(3-y)^2-4(1-3y)\geq 0 \Rightarrow 9-6y+y^2-4+12y\geq 0 \Rightarrow y^2+6y+5\geq 0 \Rightarrow (y+1)(y+5)\geq 0$

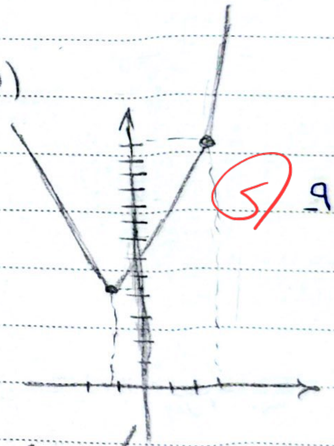
$y=-1, -5$ $\xrightarrow{\text{اما } x^2\geq 0}$ $R_f=[\frac{1}{3}, +\infty)$

ب) $y=\frac{x^2+5}{\sqrt{x^2+4}}$ $x^2+4\geq 0 \Rightarrow x\in \mathbb{R}$ $R_f=[\frac{5}{2}, +\infty)$

$x^2\geq 0 \Rightarrow x=0 \Rightarrow \sqrt{4}=2$

$\frac{x^2+5}{\sqrt{x^2+4}}=\frac{(x^2+4)+1}{\sqrt{x^2+4}}=\frac{x^2+4}{\sqrt{x^2+4}}+\frac{1}{\sqrt{x^2+4}}=\sqrt{x^2+4}+\frac{1}{\sqrt{x^2+4}}$ راه ۲

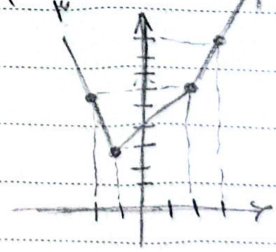
$a=\sqrt{x^2+4}\geq 2 \Rightarrow 2+\frac{1}{2}=\frac{5}{2}=2/5 \Rightarrow R_f=[2/5, +\infty)$



۹- (۲)

$y=|x-3|+|3x+4|$ (3 ، $-\frac{4}{3}$)

$y=\begin{cases} x-3+3x+4 & x>3 \\ -x+3+3x+4 & -\frac{4}{3}\le x\le 3 \\ -x+3-3x-4 & x<-\frac{4}{3} \end{cases} \Rightarrow \begin{cases} 4x+1 & x\geq 3 \\ 2x+7 & -\frac{4}{3}\le x<3 \\ -4x-1 & x<-\frac{4}{3} \end{cases}$



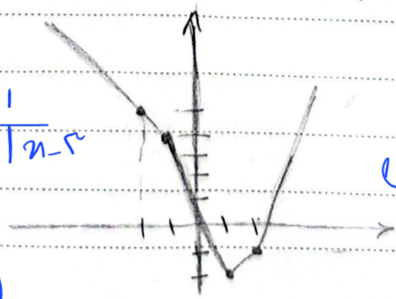
۱۰- (۱)

الف) $y=|x-2|+|2x+2|$ (2 ، -1)

-1		2	
$-3x$	$x+4$		$3x$

2	3	-1	-2
6	9	3	9

$R=[3, +\infty)$



ب) $y=|2x-2|-|x+1|$ ($+1$ ، -1)

-1		1	
$-x+3$	$-3x+1$		$x-3$

$+1$	$+2$	-1	-2
-2	-1	4	$+5$

نوشتن برد و ضابطه بندی

$R=[-2, +\infty)$