

نقطه بحرانی

نقطه بازدهی

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الف) $y = x^3 - 3x^2 + 3x + 1 \rightarrow y = (x-1)^3 + 1 \Rightarrow (x-1)^3 = y-1$

$\Rightarrow x = \sqrt[3]{y-1} + 1 \Rightarrow R_y = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x} \Rightarrow yx^2 - 2yx - 1 = 0 \Rightarrow x = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y} \Rightarrow y \neq 0$

$\Rightarrow f(y) = y(y+1) \geq 0 \Rightarrow y \in (-\infty, -1] \cup (0, +\infty)$

الف) $y = x^2 - 4x + 10 \rightarrow \min = \begin{cases} \frac{-b}{2a} = \frac{4}{2} = 2 \\ f(2) = 10 - 16 + 10 = 4 \end{cases} \Rightarrow R_y = [4, +\infty)$

ب) $y = -x^2 + 4x + 3 \rightarrow \max = \begin{cases} \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ f(2) = -4 + 16 + 3 = 15 \end{cases} \Rightarrow R_y = (-\infty, 15]$

ج) $y = \sqrt{x^2 - 4x - 3} \rightarrow \min = \begin{cases} \frac{-b}{2a} = 2 \\ f(2) = -11 = -V \end{cases} \Rightarrow R = \sqrt{[-V, +\infty)} = [0, +\infty)$

د) $y = \sqrt{4x - x^2} \rightarrow \max = \begin{cases} \frac{-b}{2a} = 2 \\ f(2) = 4 = V \end{cases} \Rightarrow R = \sqrt{(-\infty, V]} = [0, 2]$

الف) $y = x^3 - 3x^2 + 3x + 1 \rightarrow R_y = \mathbb{R}$

ب) $y = x^3 - 4x^2 + 4x + 2 \rightarrow R_y = \mathbb{R}$

ج) $y = \sqrt{x^3 - 4x^2 + 4x + 1} \rightarrow R = \sqrt{R} = [0, +\infty)$

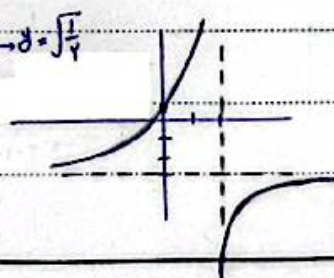
د) $y = (x^3 - 4x^2 + 4x + 1)^2 \rightarrow R_y = \mathbb{R}^+ \Rightarrow [0, +\infty)$

الف) $y = \frac{3x+1}{x-2} \rightarrow R_y = \mathbb{R} - \left\{\frac{a}{c}\right\} = \mathbb{R} - \left\{\frac{3}{2}\right\}$

ب) $y = \frac{2x+1}{x+1} \rightarrow R_y = \mathbb{R} - \left\{\frac{a}{c}\right\} = \mathbb{R} - \left\{\frac{2}{1}\right\}$

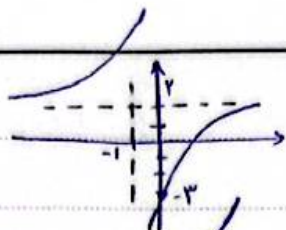
الف) $y = \sqrt{\frac{2x+1}{x+1}} \rightarrow R_y = \sqrt{\mathbb{R} - \left\{\frac{a}{c}\right\}} = [0, +\infty) - \left\{\sqrt{\frac{2}{1}}\right\}$

ب) $y = \sqrt{\frac{2x+1}{-x+2}} \Rightarrow x \rightarrow 2 \Rightarrow y = \sqrt{\frac{5}{0}} \Rightarrow R_y = [0, +\infty)$



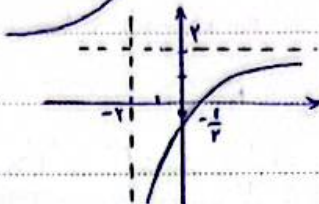
نقطه بحرانی: $\frac{a}{c}$
نقطه بازدهی: $\frac{a}{c}$

ا) $y = \frac{x^2 - 3}{x + 1} \rightarrow \text{O.P.} = -1$
 $x = 0 \rightarrow -3$
 $1, 2 = 2$



4. جانب قائم = منبج (O.P.)
 $(1, 2) \frac{a}{c}$

ب) $y = \frac{x^2 - 1}{x + 2} \rightarrow \text{O.P.} = -2$
 $x = 0 \rightarrow -\frac{1}{2}$
 $1, 2 = 2$



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ا) $\sin x + \frac{1}{\sin x} \rightarrow (-\infty, -2] \cup [2, +\infty)$

ب) $\frac{x^4 + 1}{x^2} = x^2 + \frac{1}{x^2} \rightarrow (-\infty, 2] \cup [2, +\infty)$

ج) $\frac{\sqrt[3]{x^3 + 1}}{\sqrt[3]{x}} = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}} \rightarrow (-\infty, -2] \cup [2, +\infty)$

د) $\sqrt{x} + \frac{1}{\sqrt{x}} \rightarrow [2, +\infty)$

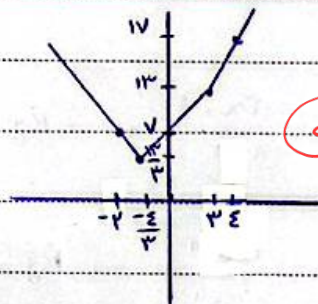
ا) $y = \frac{x^2}{x^2 + 3} \rightarrow \frac{x^2 - 3}{x^2 + 3} \rightarrow \left(\frac{1}{3}, +\infty\right)$
 $\min(x^2 + 3) = 3$
 $\sqrt[3]{3 + \frac{1}{3}} \rightarrow +\infty$

ب) $\frac{x^2 + \Delta}{\sqrt{x^2 + \Delta}} = \sqrt{x^2 + \Delta} + \frac{1}{\sqrt{x^2 + \Delta}} \rightarrow \min(x^2 + \Delta) = \Delta \Rightarrow y = \sqrt{\Delta} + \frac{1}{\sqrt{\Delta}} \Rightarrow \frac{a}{y}$
 $\Rightarrow R_y = \left[\frac{a}{y}, +\infty\right)$

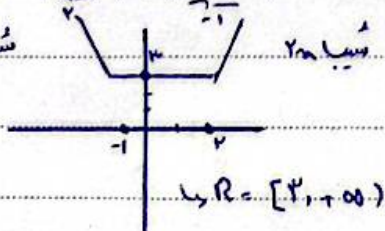
$y = \frac{|x - 3|}{2} + \frac{|x + 3|}{2} \Rightarrow$
 $\frac{3 - x - \varepsilon - 3 + x}{2} = \frac{3 - x + 3 + x + \varepsilon}{2} \Rightarrow \frac{6 - \varepsilon}{2} = \frac{6 + \varepsilon}{2} \Rightarrow 3 - \frac{\varepsilon}{2} = 3 + \frac{\varepsilon}{2} \Rightarrow \varepsilon = 0$

if $x = \varepsilon \rightarrow y = 14$

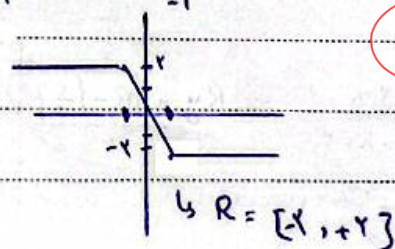
if $x = -\varepsilon \rightarrow y = 14$



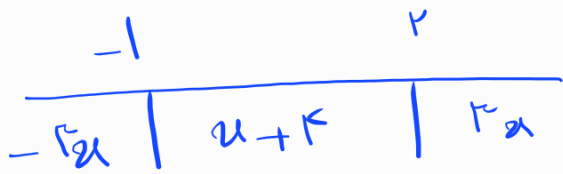
ا) $y = \frac{|x - 1|}{2} + \frac{|x + 1|}{2} \rightarrow \text{O.P.} = 1$



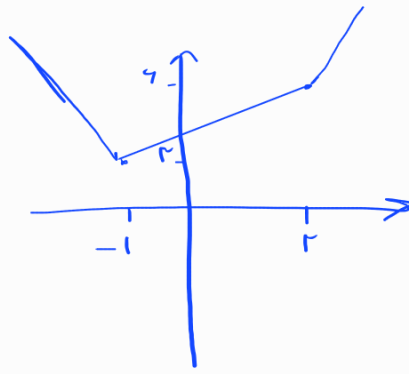
ب) $y = \frac{|x - 2|}{2} - \frac{|x + 1|}{2} \rightarrow \text{O.P.} = -1$



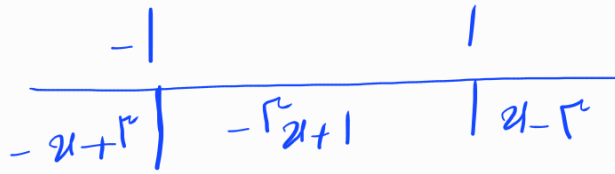
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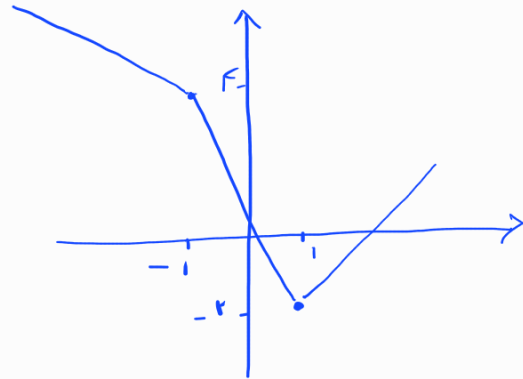
$$R = [r, +\infty)$$



(a) (b)



$$R = [-1, +\infty)$$



(b) (a)