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۱) $y = x^2 - 2x + 1 \Rightarrow y = (x-1)^2 + 1 \Rightarrow (x-1)^2 = y-1 \Rightarrow x = \sqrt{y-1} + 1 \Rightarrow R_f = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x} \Rightarrow yx^2 - 2xy = 1 \Rightarrow yx^2 - 2xy - 1 = 0 \Rightarrow yx(x-2) - 1 = 0 \Rightarrow x(x-2) = \frac{1}{y} \Rightarrow x^2 - 2x = \frac{1}{y} \quad y \neq (-1, 0) \quad R_y = (-\infty, -1] \cup (0, +\infty)$

الف) $x^2 - 4x + 1$
 ا) $x = \frac{-b}{2a} = \frac{4}{2} = 2 \quad y = 2^2 - 4 \cdot 2 + 1 = -7 \quad R_y = [-7, +\infty)$

ب) $-x^2 + 4x + 3$
 ا) $x = \frac{-b}{2a} = \frac{-4}{-2} = 2 \quad y = -2^2 + 4 \cdot 2 + 3 = 11 \quad R_y = (-\infty, 11]$

ج) $\sqrt{x^2 - 4x - 3}$
 ا) $x = \frac{-b}{2a} = \frac{4}{2} = 2 \quad y = 2^2 - 4 \cdot 2 - 3 = -7 \quad R_y = [-7, +\infty) = [0, +\infty)$

د) $\sqrt{4x - x^2}$
 ا) $x = \frac{-b}{2a} = \frac{-4}{-2} = 2 \quad y = 4 - 4 = 0 \quad R_y = [0, 4]$

الف) $y = x^2 - 5x^2 + 2x + 1 \rightarrow R_y = \mathbb{R}$ ب) $x^2 - 4x^2 + 2x + 2 \rightarrow R_y = \mathbb{R}$

ج) $y = \sqrt{x^2 - 4x^2 + 5x + 1} \rightarrow R_y = \sqrt{\mathbb{R}} = [0, +\infty)$ د) $(x^2 - 4x^2 + 2x + 1)^2 \rightarrow R_y = [0, +\infty)$

الف) $\frac{x+1}{x-2} \rightarrow R_y = \mathbb{R} - \{2\}$ ب) $y = \frac{x+1}{x+1} \rightarrow R_y = \mathbb{R} - \{2\}$

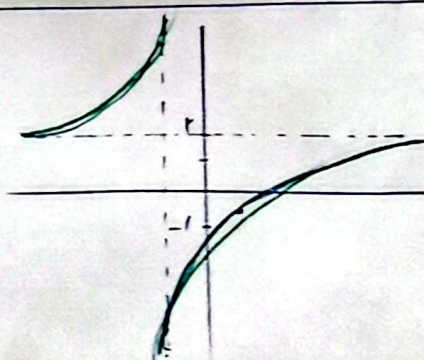
الف) $y = \sqrt{\frac{x+1}{x+1}} \quad R_y = [0, +\infty) - \{\sqrt{2}\}$ ب) $y = \sqrt{\frac{x+1}{x+1}} \rightarrow R_y = \sqrt{\mathbb{R} - \{2\}} = [0, +\infty)$

الف) $y = \frac{x-2}{x+1}$

محاسبه $x = -1$

محاسبه $y = 2$

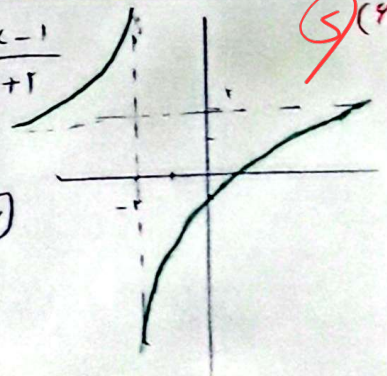
نقطه $(-1, 2)$



ب) $y = \frac{x-1}{x+1}$

محاسبه $x = -1$
 محاسبه $y = 2$

نقطه $(-1, 2)$



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الف) $y = \sin x + \frac{1}{\sin x} \rightarrow R_y = (-\infty, -1] \cup [1, +\infty)$

ب) $y = \frac{x^2+1}{x^2} \rightarrow x^2 + \frac{1}{x^2} = R_y = (-\infty, -1] \cup [1, +\infty)$

ج) $y = \frac{\sqrt{x^2+1}}{\sqrt{x}} = \sqrt{x} + \frac{1}{\sqrt{x}} = R_y = (-\infty, -1] \cup [1, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} = R_y = [1, +\infty)$

الف) $y = x^2 + \frac{1}{x^2+2} \quad x^2 = t \rightarrow t \in [0, +\infty)$

$\frac{1}{t+2} + t = y \quad t=0 \rightarrow \frac{1}{2} + 0 = \frac{1}{2}$

ب) $y = \frac{x^2+2}{\sqrt{x^2+2}} \quad x^2 = t \rightarrow t \in [0, +\infty)$
 $\frac{t+2}{\sqrt{t+2}} = y \rightarrow \frac{1}{\sqrt{t+2}} + \sqrt{t+2} = y$

$t=1 \rightarrow \frac{1}{1+1} + 1 = 1, 2$
 $t=2 \rightarrow \frac{1}{2+2} + 2 = 2, 1$

$\frac{t+2}{\sqrt{t+2}} = \frac{(t+2)+1}{\sqrt{t+2}} = \frac{t+2}{\sqrt{t+2}} + \frac{1}{\sqrt{t+2}} = \frac{1}{\sqrt{t+2}} + \sqrt{t+2}$

$\frac{t+2}{\sqrt{t+2}} = \frac{(t+2)+1}{\sqrt{t+2}} = \frac{t+2}{\sqrt{t+2}} + \frac{1}{\sqrt{t+2}} = \frac{1}{\sqrt{t+2}} + \sqrt{t+2}$

$2 \leq t+2 \rightarrow 2 \leq \sqrt{t+2} \quad 2 \leq \frac{1}{\sqrt{t+2}} + \sqrt{t+2}$

$R_y = [\frac{5}{4}, +\infty)$

$y = |x-2| + |2x+1|$

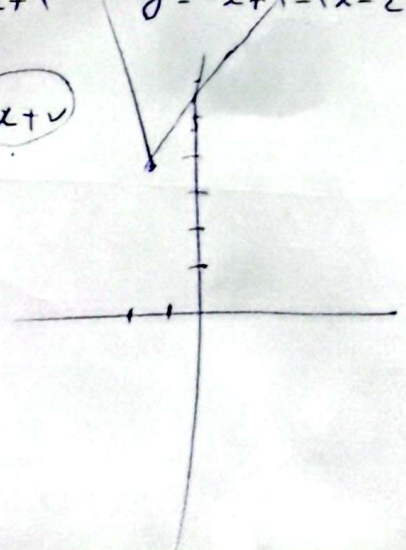
$x-2=0 \rightarrow x=2$
 $2x+1=0 \rightarrow x=-\frac{1}{2}$

① $x < -\frac{1}{2}$
 $|x-2| = -(x-2) = -x+2$
 $|2x+1| = -(2x+1) = -2x-1$
 $y = -x+2-2x-1 = -3x+1$

② $-\frac{1}{2} \leq x < 2$
 $|x-2| = 2-x$
 $|2x+1| = 2x+1$
 $y = 2-x+2x+1 = x+3$

③ $x \geq 2$
 $|x-2| = x-2$
 $|2x+1| = 2x+1$
 $y = x-2+2x+1 = 3x-1$

$y = \begin{cases} -3x+1 & x < -\frac{1}{2} \\ x+3 & -\frac{1}{2} \leq x < 2 \\ 3x-1 & x \geq 2 \end{cases}$



$$a) y = |x-2| + |x+2|$$

$$x-2=0 \rightarrow x=2$$

$$x+2=0 \rightarrow x=-2 \rightarrow x=-1$$

$$① \text{ no } i \rightarrow -1 > x \quad |x-2| = -(x-2) = -x+2 \quad (1)$$

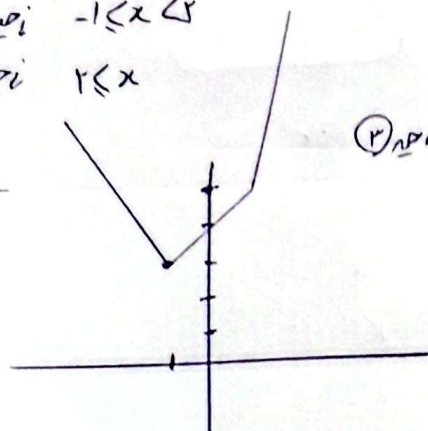
$$|x+2| = -(x+2) = -x-2$$

$$y = -x+2 - x-2 = -2x$$

$$② \text{ no } i \rightarrow -1 \leq x < 2$$

$$③ \text{ no } i \rightarrow x \geq 2$$

$$④ \text{ no } i \rightarrow x \geq 2$$



$$① \text{ no } i \rightarrow 1 \leq x < 2 \quad |x-2| = -(x-2) = -x+2$$

$$|x+2| = x+2$$

$$y = -x+2 + x+2 = 4$$

$$② \text{ no } i \rightarrow x \leq -1 \quad |x-2| = x-2$$

$$|x+2| = x+2$$

$$y = x-2 + x+2 = 2x$$

$$Ry = [2, +\infty)$$

$$y = \begin{cases} -2x & x < -1 \\ 4 & -1 \leq x < 2 \\ 2x & x \geq 2 \end{cases}$$

$$b) y = |x-2| - |x+1|$$

$$x-2=0 \rightarrow x=2$$

$$x+1=0 \rightarrow x=-1$$

$$① \text{ no } i \rightarrow x < -1$$

$$② \text{ no } i \rightarrow -1 \leq x < 2$$

$$③ \text{ no } i \rightarrow x \geq 2$$

$$① \text{ no } i \rightarrow x < -1 \quad |x-2| = -(x-2) = -x+2$$

$$|x+1| = -(x+1) = -x-1$$

$$y = (-x+2) - (-x-1) = 3$$

$$② \text{ no } i \rightarrow -1 \leq x < 2 \quad |x-2| = -(x-2) = -x+2$$

$$|x+1| = x+1$$

$$y = (-x+2) - (x+1) = -2x+1$$

$$③ \text{ no } i \rightarrow x \geq 2$$

$$|x-2| = x-2$$

$$|x+1| = x+1$$

$$y = (x-2) - (x+1) = -3$$

$$y = \begin{cases} 3 & x < -1 \\ -2x+1 & -1 \leq x < 2 \\ -3 & x \geq 2 \end{cases}$$

$$Ry = [-3, +\infty)$$

