

الف) $y = x^3 - 3x^2 + 3x$

$(x-1)^3 = x^3 - 3x^2 + 3x - 1$

$y = (x-1)^3 + 1$

$y-1 = (x-1)^3$

$x-1 = \sqrt[3]{y-1}$

$x = \sqrt[3]{y-1} + 1 \Rightarrow R_f = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x}$

$y(x^2 - 2x) = 1$

$y(x^2 - 2x) - 1 = 0 \Rightarrow x = \frac{2 \pm \sqrt{4y^2 + 4y}}{2}$

$\Delta = 4y^2 + 4y$

$y(y+1) \geq 0$

$R_f = (-\infty, -1] \cup [0, +\infty)$

الف) $y = x^2 - 4x + 1$

$\min = \frac{\Delta}{-4a} = \frac{14 - 16}{-4} = \frac{-2}{-4} = \frac{1}{2}$

$R_f = [\frac{1}{2}, +\infty)$

ب) $y = -x^2 + 4x + 12$

$\max = \frac{\Delta}{-4a} = \frac{16 + 12}{-4} = \frac{28}{-4} = -7$

$R_f = (-\infty, 12]$

ج) $y = \sqrt{x^2 - 4x - 12}$

$\min = \frac{\Delta}{-4a} = \frac{14 + 12}{-4} = \frac{26}{-4} = -\frac{13}{2}$

$R_f = [-\frac{13}{2}, +\infty)$

د) $y = \sqrt{4x - x^2}$

$\max = \frac{\Delta}{-4a} = \frac{16}{-4} = -4$

$R_f = [0, 4]$

الف) $y = x^3 - 3x^2 + 3x + 1$

$R_f = \mathbb{R}$

ب) $y = \frac{x^3 - 4x^2 + 12x + 1}{x^2 - 2x}$

$R_f = \mathbb{R}$

ج) $y = \sqrt{x^3 - 4x^2 + 12x + 1}$

$R_f = [0, +\infty)$

د) $y = (x^3 - 4x^2 + 12x + 1)^{\frac{1}{2}}$

$R_f = [0, +\infty)$

الف) $y = \frac{3x+1}{x-2} \xrightarrow{\text{عکس‌المنعکس}} R_f = \mathbb{R} - \{3\}$

$\frac{1}{x-2} \rightarrow yx - y = 3x + 1$

$yx - 3x = y + 1$

$x(y-3) = y+1 \Rightarrow x = \frac{y+1}{y-3}$

$R_f = \mathbb{R} - \{3\}$

ب) $y = \frac{3x+1}{x+1} \xrightarrow{\text{عکس‌المنعکس}} R_f = \mathbb{R} - \{2\}$

$\frac{1}{y-2} \rightarrow yx + y = 3x + 1 \Rightarrow x(y-3) = 1-y$

$x = \frac{1-y}{y-3}$

$R_f = \mathbb{R} - \{2\}$

الف) $y = \sqrt{\frac{3x+1}{x+1}}$

$yx + y = 3x + 1$

$x(y-3) = 1-y$

$x = \frac{1-y}{y-3} \rightarrow R_f = \mathbb{R} - \{2\}$

$R_f = (0, +\infty) - \{2\}$

$R_f = \mathbb{R} - \{2\}$

$R_f = (0, +\infty) - \{2\}$

ب) $y = \sqrt{\frac{3x+1}{x-2}}$

$R_f = [0, +\infty)$

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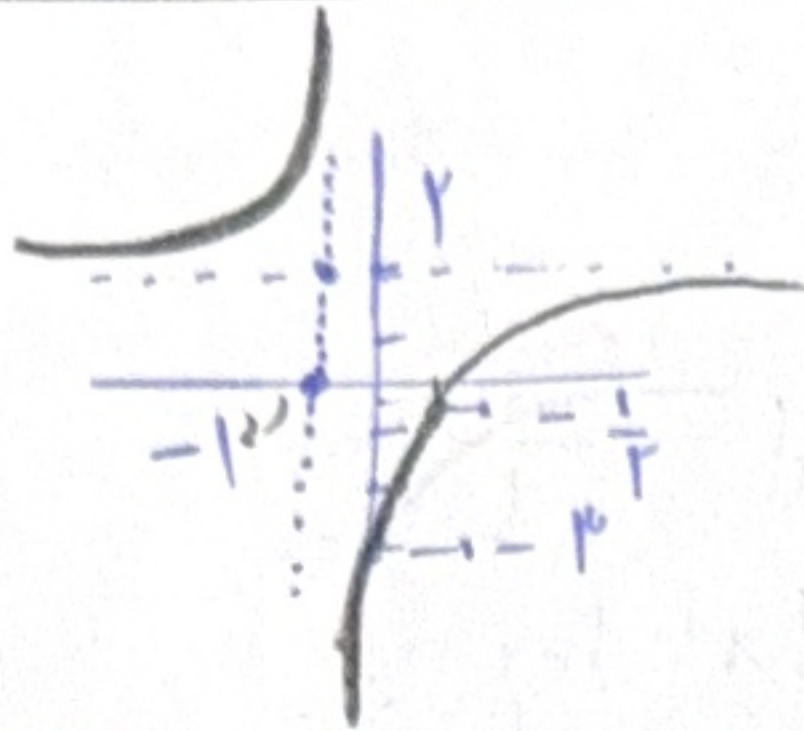
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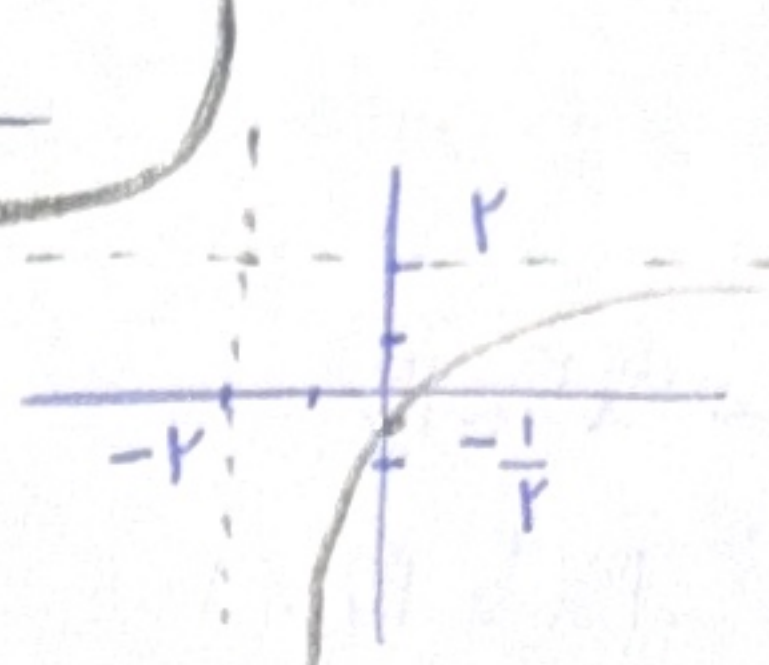
الف) $y = \frac{rx - r}{x+1}$

ملاحظات: $\rightarrow -1$
 ملاحظات: $\rightarrow \frac{r}{1} = r$
 if $x=0 \Rightarrow y = \frac{-r}{1} = -r$
 if $x=1 \Rightarrow \frac{r-r}{1+1} = \frac{-1}{r}$



ب) $y = \frac{rx - 1}{x+r}$

ملاحظات: $\rightarrow -r$
 ملاحظات: $\rightarrow r$
 if $x=0 \Rightarrow y = \frac{-1}{r}$



الف) $y = \sin x + \frac{1}{\sin x} \Rightarrow R_f = (-\infty, -r] \cup [r, +\infty)$ $\rightarrow y = \frac{x^4+1}{x^2} \rightarrow y = x^2 + \frac{1}{x^2}$
 $R_f = (-\infty, -r] \cup [r, +\infty)$

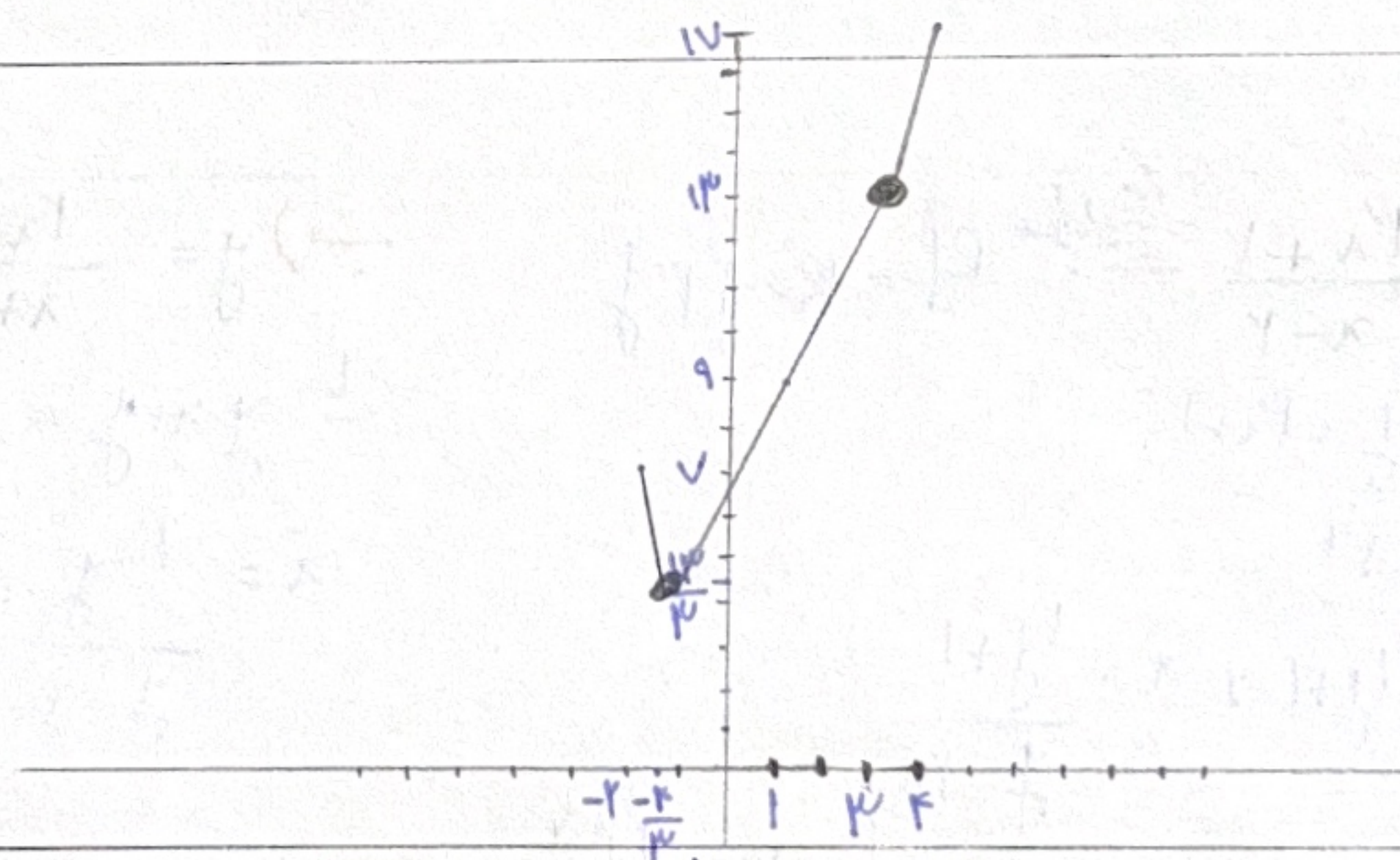
ج) $y = \frac{\sqrt[n]{x^r+1}}{\sqrt[n]{x}} \rightarrow \sqrt[n]{x} + \frac{1}{\sqrt[n]{x}} \rightarrow R_f = (-\infty, -r] \cup [r, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \rightarrow R_f = [r, +\infty)$

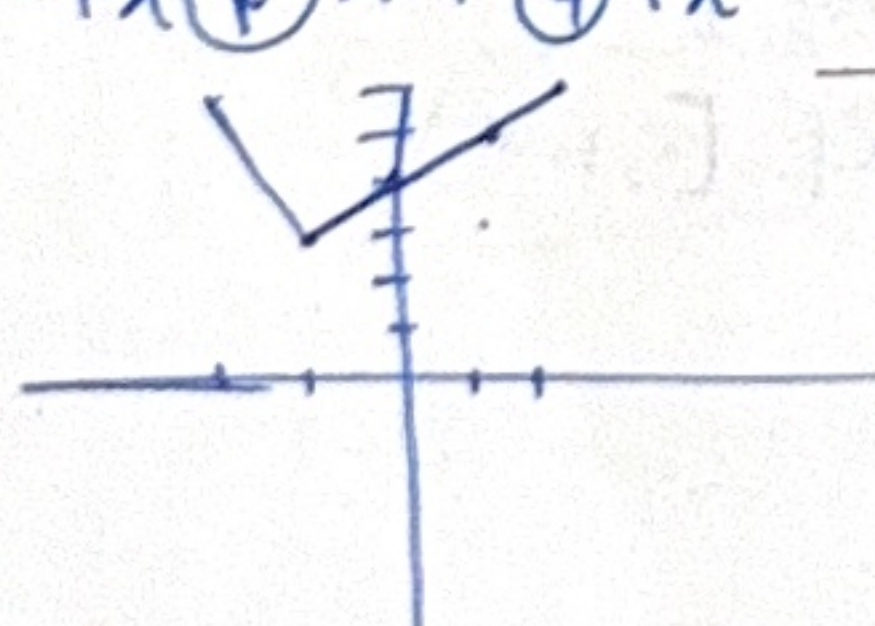
الف) $y = x^r + \frac{1}{x^r+r} =$
 $x^r + \frac{1}{x^r+r} + r - r \rightarrow \frac{x^r+r+1}{x^r+r} - r$
 if $x=0 \Rightarrow \frac{r+1}{r} = \frac{1}{r}$
 $\frac{1}{r} - r = 1 - \frac{r}{r} = \frac{1}{r}$
 $R_f = [\frac{1}{r}, +\infty)$
 ملاحظات: $\rightarrow$ حد أدنى

ب) $y = \frac{x^r+0}{\sqrt{x^r+r}} \rightarrow \frac{x^r+r+1}{\sqrt{x^r+r}}$
 $y = \frac{x^r+r}{\sqrt{x^r+r}} + \frac{1}{\sqrt{x^r+r}}$
 $y = \sqrt{x^r+r} + \frac{1}{\sqrt{x^r+r}}$
 if $x=0 \Rightarrow y = \sqrt{r} + \frac{1}{\sqrt{r}} = r + \frac{1}{r} = \frac{r^2+1}{r} \Rightarrow R_f = [\frac{r^2+1}{r}, +\infty)$

$y = |x-r| + |rx+r|$
 $\begin{matrix} r & & -r \\ -\frac{r}{r} & & r \end{matrix}$
 $-x+r \quad x-r \quad x-r+rx+r$
 $-rx-1 \quad rx+r \quad rx+1$
 if $x=1 \Rightarrow y=9$
 if $x=-1 \Rightarrow y=9$
 $y=0$



الف) $y = |x-r| + |rx+r|$
 $\begin{matrix} r & & -1 \\ -1 & & r \end{matrix}$
 $-rx+r \quad x+r \quad rx$
 $R_f = [r, +\infty)$



ب) $y = |rx-r| - |x+1|$
 $\begin{matrix} 1 & & -1 \\ -1 & & 1 \end{matrix}$
 $-x+r \quad -rx+1 \quad x-r$
 $R_f = [-r, +\infty)$

