

الف) $x^4 - 1^2 x^2 + 1^2 x + 1 - 1 = (x-1)^2 + 1 \rightarrow y = (x-1)^2 + 1$ $\Rightarrow y-1 = (x-1)^2 \Rightarrow \sqrt{y-1} = x-1 \rightarrow x = \sqrt{y-1} + 1 \Rightarrow R_f = \mathbb{R}$ ب) $\frac{1}{x^2-1} = y \Rightarrow yx^2 - 1yx = 1 \Rightarrow yx^2 - 2xy - 1 = 0 \Rightarrow \Delta \geq 0$ $\Delta = b^2 - 4ac = (-2y)^2 - 4(y)(-1) = 4y^2 + 4y \rightarrow 4y^2 + 4y \geq 0 \rightarrow y^2 + y \geq 0 \Rightarrow y \leq -1 \text{ or } y \geq 0$ $R_f = (-\infty, -1] \cup (0, +\infty)$	1
الف) $\frac{-b}{2a} = \frac{f}{2} = 2 \rightarrow y = (2)^2 - f(2) + 1 = 4 - 1 + 1 = 4, a > 0 \Rightarrow R_f = [4, +\infty)$ ب) $\frac{-b}{2a} = \frac{-4}{-2} = 2 \rightarrow y = -(2)^2 + 4(2) + 2 = -4 + 8 + 2 = 6, a < 0 \Rightarrow R_f = (-\infty, 6]$ ج) $\frac{-b}{2a} = \frac{f}{2} = 2 \rightarrow y = (2)^2 - f(2) - 3 = 4 - 1 - 3 = 0, a > 0 \Rightarrow \sqrt{[-1, +\infty)} = [0, +\infty) = R_f$ د) $\frac{-b}{2a} = \frac{-4}{-2} = 2 \rightarrow y = 4(2) - (2)^2 = 8 - 4 = 4, a < 0 \Rightarrow \sqrt{(-\infty, 4]} = [0, 2] = R_f$	2
الف) $\Rightarrow R_f = \mathbb{R}$ ب) $\Rightarrow R_f = \mathbb{R}$ ج) $\Rightarrow \sqrt{\mathbb{R}} = [0, +\infty) = R_f$ د) $\Rightarrow R_f = [0, +\infty)$	3
الف) $\Rightarrow R_f = \mathbb{R} - \left\{ \frac{a}{c} \right\} = \mathbb{R} - \{2\}$ ب) $\Rightarrow R_f = \mathbb{R} - \left\{ \frac{a}{c} \right\} = \mathbb{R} - \{2\}$	4
الف) $\frac{2x+4}{x+1} \Rightarrow R = \mathbb{R} - \{2\} \Rightarrow \sqrt{\mathbb{R} - \{2\}} = [0, +\infty) - \{\sqrt{2}\} = R_f$ ب) $\frac{2x+1}{1-x} \Rightarrow R = \mathbb{R} - \{-2\} \Rightarrow \sqrt{\mathbb{R} - \{-2\}} = [0, +\infty) = R_f$	5

الف) $\begin{cases} \text{موجب قائم} \Rightarrow x+1=0 \Rightarrow x=-1 \\ \text{موجب اقتر} = \frac{a}{c} = 1 \end{cases}$

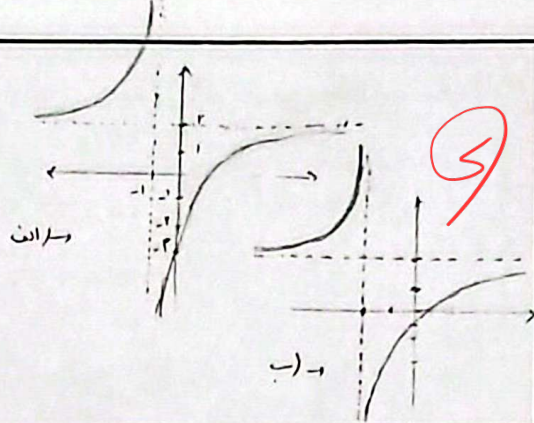
$x=0 \rightarrow y=-3$

ب) $\begin{cases} \text{موجب قائم} \rightarrow x+2=0 \rightarrow x=-2 \\ \text{موجب اقتر} = \frac{a}{c} = 2 \end{cases}$

ج) $\begin{cases} \text{موجب قائم} \rightarrow x+2=0 \rightarrow x=-2 \\ \text{موجب اقتر} = \frac{a}{c} = 2 \end{cases}$

$x=0 \rightarrow y = -\frac{1}{2}$

مراتب



٦

الف) $R_f = (-\infty, -2] \cup [2, +\infty) \rightarrow (\sin x > 0 \text{ و } \sin x < 0)$

ب) $\frac{x^4+1}{x^3} = x + \frac{1}{x^3} \Rightarrow R_f = (-\infty, -2] \cup [2, +\infty) \rightarrow (x^3 > 0 \text{ و } x^3 < 0)$

ج) $\sqrt{x} + \frac{1}{\sqrt{x}} \Rightarrow R_f = (-\infty, -2] \cup [2, +\infty) \rightarrow (\sqrt{x} > 0 \text{ و } \sqrt{x} < 0)$

د) $\sqrt{x} + \frac{1}{\sqrt{x}} \Rightarrow R_f = [2, +\infty) \rightarrow (\sqrt{x} > 0)$

٧

الف) $\text{بردار} : x^2 + \frac{1}{x^2} : \text{مشتق} = 2x - \frac{2}{x^3} = \frac{2x^4 - 2}{x^3} = \frac{2(x^4 - 1)}{x^3} = \frac{2(x^2 - 1)(x^2 + 1)}{x^3} = \frac{2(x-1)(x+1)(x^2 + 1)}{x^3}$

$R_f = \text{بردار} : (x^2 + \frac{1}{x^2}) - 1 \Rightarrow \frac{10}{3} - 1 = \frac{7}{3} \rightarrow R_f = [\frac{7}{3}, +\infty)$

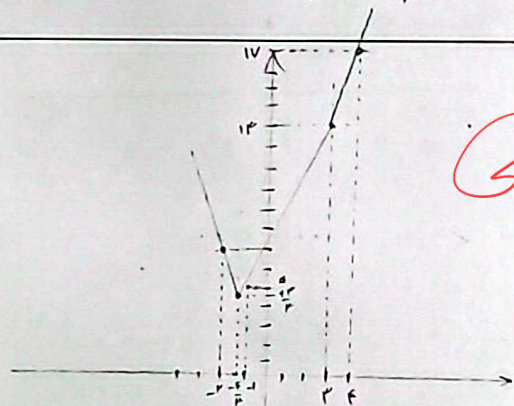
ب) $\frac{x^2 + 1}{\sqrt{x^2 + 4}} = \sqrt{x^2 + 4} + \frac{1}{\sqrt{x^2 + 4}} \rightarrow \text{مشتق} = 2x + \frac{-x}{(x^2 + 4)^{3/2}} = \frac{2x^3 + x}{(x^2 + 4)^{3/2}} = \frac{x(2x^2 + 1)}{(x^2 + 4)^{3/2}}$

$(\sqrt{x^2 + 4} > 0)$

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$\begin{cases} x - 1 + 3x + 6 = 4x + 1 & x > 3 \\ 1 - x + 3x + 6 = 2x + 7 & -\frac{1}{3} \leq x \leq 3 \\ 1 - x - 1 - 3x = -4x & x < -\frac{1}{3} \end{cases}$

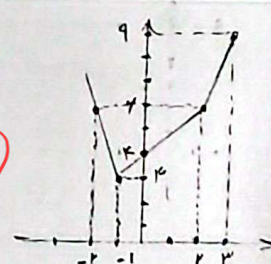
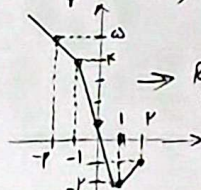
x	3	4	$-\frac{1}{3}$	-1	-2
y	13	10	$\frac{10}{3}$	5	4



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الف) $\begin{cases} x - 2 + 2x + 2 = 3x & x > 2 \\ 2x + 2 + 2 - x = x + 4 & -1 \leq x \leq 2 \\ 2 - x - 2x - 2 = -3x & x < -1 \end{cases} \Rightarrow R_f = [2, +\infty)$

ب) $\begin{cases} 2x - 2 - x - 1 = x - 3 & x > 1 \\ 2 - 2x - x - 1 = -3x + 1 & -1 \leq x \leq 1 \\ 2 - 2x + x + 1 = 3 - x & x < -1 \end{cases} \Rightarrow R_f = [-1, +\infty)$



x	2	1	0	-1	-2
y	4	3	2	1	0

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