

سوال 1:

الف) $y = x^3 - 3x^2 + 3x$

$y = (x-1)^3 + 1 \rightarrow y-1 = (x-1)^3 \rightarrow x = \sqrt[3]{y-1} + 1 \quad R_f = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x}$

$y(x^2 - 2x) = 1 \rightarrow y(x^2 - 2x) = 1 \rightarrow x^2 - 2x = \frac{1}{y} \rightarrow (x-1)^2 - 1 = \frac{1}{y} \rightarrow x-1 = \sqrt{\frac{1}{y} + 1} \rightarrow x = \sqrt{\frac{1}{y} + 1} + 1$

$\frac{1+y}{y} \geq 0 \quad \begin{array}{c|cc} & -1 & 0 \\ \hline & + & - & + \end{array} \quad R_f = (-\infty, -1] \cup (0, +\infty)$

سوال 2:

الف) $y = x^2 - 4x + 10 \quad a > 0$

$\begin{cases} x_{min} = \frac{-b}{2a} \\ y_{min} = (2)^2 - 4(2) + 10 = 6 \end{cases} \quad R_f = [6, +\infty)$

ب) $y = -x^2 + 4x + 3 \quad a < 0$

$\begin{cases} x_{max} = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y_{max} = (2)^2 - 4(2) + 3 = 1 \end{cases} \quad R_f = (-\infty, 1]$

ج) $y = \sqrt{x^2 - 4x - 3} \quad a > 0$

$\begin{cases} x_{min} = \frac{4}{2} = 2 \\ y_{min} = (2)^2 - 4(2) - 3 = -7 \end{cases} \rightarrow R_f = \sqrt{[-7, +\infty)} = [0, +\infty)$

د) $y = \sqrt{4x - x^2} \quad a < 0$

$\begin{cases} x_{max} = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y_{max} = 2 \end{cases} \quad R_f = \sqrt{[-\infty, 4]} = [0, 2]$

سوال 3:

الف) $y = x^3 - 3x^2 + 3x + 1 \quad \text{توان فرد} \quad R_f = \mathbb{R}$

ب) $y = x^4 - 4x^2 + 3 \quad \text{توان زوج} \quad R_f = \mathbb{R}$

ج) $y = \sqrt{x^4 - 4x^2 + 3} \quad \text{دیر دایره ای} \quad R_f = \sqrt{\mathbb{R}} = [0, +\infty)$

د) $y = (x^3 - 3x^2 + 3x + 1)^2 \quad \text{توان فرد} \quad R_f = (-\infty, +\infty)^2 \rightarrow R_f = [0, +\infty)$

سوال 4:

الف) $y = \frac{x+1}{x-2} \quad R_f = \mathbb{R} - \{2\}$

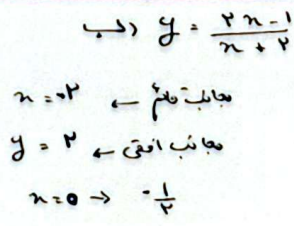
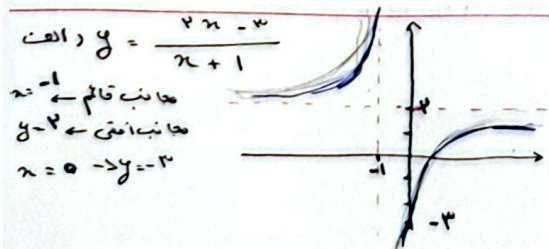
ب) $y = \frac{x+1}{x+2} \quad R_f = \mathbb{R} - \{-2\}$

سوال 5:

الف) $y = \sqrt{\frac{x+1}{x+2}} \quad \text{هذیر دایره ای} \quad R_f = \sqrt{\mathbb{R} - \{-2\}} = R_f = [0, +\infty) - \{\sqrt{2}\}$

ب) $y = \sqrt{\frac{x+1}{x-2}} \quad \text{هذیر دایره ای} \quad R_f = \sqrt{\mathbb{R} - \{2\}} = R_f = [0, +\infty)$

سوال 6:



مثال 1) $y = \sin x + \frac{1}{\sin x}$ $R_f = (-\infty, -1] \cup [1, +\infty)$

سوال 1

ب) $y = \frac{x^2+1}{x^3} = x^{-1} + \frac{1}{x^3}$ $R_f = (-\infty, -1] \cup [1, +\infty)$

ج) $y = \frac{\sqrt[3]{x^2+1}}{\sqrt{x}} = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$ $R_f = (-\infty, -1] \cup [1, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ $R_f = [1, +\infty)$

$y = x^2 + \frac{1}{x^2+x} = \frac{x^2+x}{x^2+x} - \frac{x}{x^2+x} \rightarrow (x + \frac{1}{x}), -x$

سوال 2

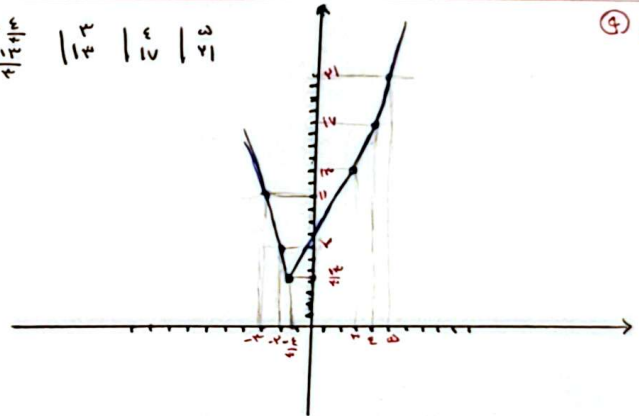
ب) $y = \frac{x^2+5}{\sqrt{x^2+4}} = \frac{x^2+4+1}{\sqrt{x^2+4}} = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}}$

ب) $y = \frac{x^2+5}{\sqrt{x^2+4}} = \frac{x^2+4+1}{\sqrt{x^2+4}} = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}}$

$y = |x-3| + |x+5|$

$-\infty \quad -3 \quad -\frac{5}{2} \quad 3 \quad 5 \quad \infty$

$-\infty$	-3	3	∞
$-x+3-x-5$	$-x+3+x+5$	$x-3+x+5$	
$-2x-2$	$2x+8$	$2x+2$	



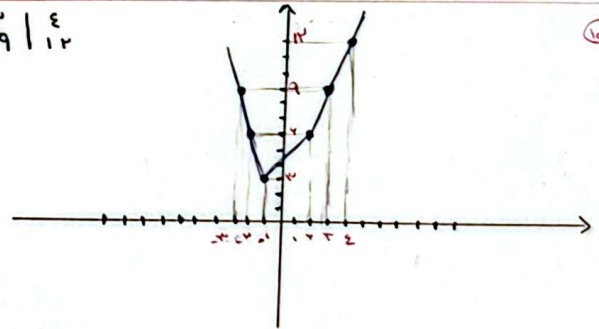
سوال 3

مثال 2) $y = |x-1| + |x+2|$

$-\infty \quad -2 \quad -1 \quad 1 \quad 2 \quad \infty$

$-\infty$	-2	1	∞
$-x+2-x-1$	$-x+2+x+1$	$x-1+x+1$	
$-2x+1$	$2x+3$	$2x$	

$R_f = [1, +\infty)$



سوال 4

ب) $y = |x-1| - |x+1|$

$-\infty \quad -1 \quad 0 \quad 1 \quad \infty$

$-\infty$	-1	1	∞
$-x+1-x-1$	$-x+1-x+1$	$x-1-x-1$	
$-2x$	$-2x+2$	$-2x-2$	

$R_f = [-1, +\infty)$

