

1) الف) $n^3 - 3n^2 + 3n + 1 - 1 = (n-1)^3 + 1 = y$ نبدأ إعطاء - 1، نأخذ الجذر

$$\sqrt[3]{y-1} = n-1$$

$$n = \sqrt[3]{y-1} + 1$$

$$R_f = \mathbb{R}$$

ب) $y = \frac{1}{n^2 - 2n} \leadsto y n^2 - 2y n = 1$

$$n^2 y - 2n y - 1 = 0 \Rightarrow \Delta = 4y^2 + 4y > 0$$

$$R_f = (-\infty, -1] \cup \left[\frac{1}{4}, +\infty \right) \cup \left[\frac{1}{4}, +\infty \right)$$

د) $y = n^2 - 5n + 1 \Rightarrow \underbrace{(n^2 - 5n + 1)}_{[0, +\infty)} + 4 \Rightarrow R_f = [4, +\infty)$ الف

$$y = -n^2 + 9n + 1 \Rightarrow \underbrace{-(n^2 - 9n + 9)}_{(-\infty, 0]} + 10 \Rightarrow R_f = (-\infty, 10]$$
 ب

$$y = \sqrt{n^2 - 5n - 1} \rightarrow (n^2 - 5n + 1) - 2 \Rightarrow \sqrt{[-2, +\infty)} = [0, +\infty)$$
 ج

$$y = \sqrt{9n - n^2} \rightarrow \frac{(-n^2 + 9n - 9)}{\underbrace{-(n^2 - 9n + 9)}_{(-\infty, 0]}} + 9 \Rightarrow \sqrt{(-\infty, 9]} = [0, 3]$$
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هـ) $y = n^3 - 3n^2 + 3n + 1 \Rightarrow \mathbb{R} = R_f \quad y = n^3 - 9n^2 + 12n + 1 \Rightarrow \mathbb{R} = R_f$

$$y = \sqrt{n^3 - 9n^2 + 12n + 1} \Rightarrow \sqrt{\mathbb{R}} = [0, +\infty) = R_f$$

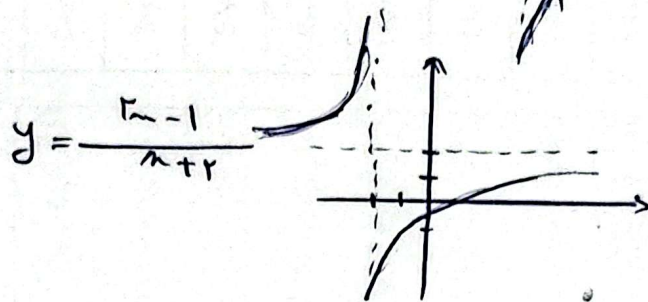
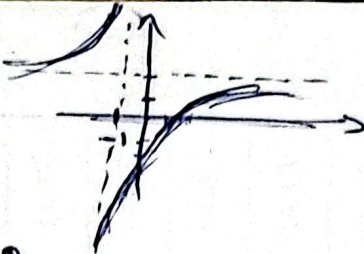
$$y = (n^3 - 9n^2 + 12n + 1)^2 \Rightarrow \mathbb{R}^+ = [0, +\infty) = R_f$$

و) $y = \frac{n+1}{n-2} \quad \mathbb{R} - \{2\} \quad y = \frac{n+1}{n+1} \quad \mathbb{R} - \{1\}$

ز) $\sqrt{\frac{n+1}{n+1}} \leadsto \frac{\sqrt{n+1}}{\sqrt{n+1}} \Rightarrow \sqrt{\mathbb{R} - \{1\}} = [0, +\infty) - \{1\}$

$$y = \sqrt{\frac{n+1}{n-2}} \Rightarrow \sqrt{\mathbb{R} - \{2\}} = [0, +\infty)$$

$$④ \quad y = \frac{r_n - r}{n+1}$$



$$⑤ \quad y = \sin n + \frac{1}{\sin n} \Rightarrow R_f = (-\infty, -r] \cup [r, +\infty)$$

$$y = n^a + \frac{1}{n^a} \Rightarrow R_f = (-\infty, -r] \cup [r, +\infty)$$

$$y = \sqrt[n]{n} + \frac{1}{\sqrt[n]{n}} \Rightarrow R_f = (-\infty, -r] \cup [r, +\infty)$$

$$y = \sqrt{n} + \frac{1}{\sqrt{n}} \Rightarrow R_f = [r, +\infty)$$

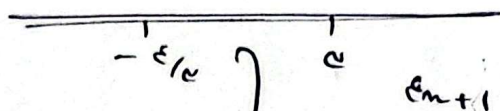
$$① \quad y = nr + \frac{1}{nr+a} \Rightarrow \underbrace{nr + \frac{1}{nr+a}}_{\min = r + \frac{1}{a} = \frac{1}{\frac{1}{a}}} - a \Rightarrow \left[\frac{1}{\frac{1}{a}}, +\infty \right)$$

$$\min = r + \frac{1}{a} = \frac{1}{\frac{1}{a}}$$

$$\frac{nr+a}{\sqrt{nr+a}} = \sqrt{nr+a} + \frac{1}{\sqrt{nr+a}} \Rightarrow \min_{n=1} \quad r + \frac{1}{r} = r_d \Rightarrow R_f = [r_d, +\infty)$$

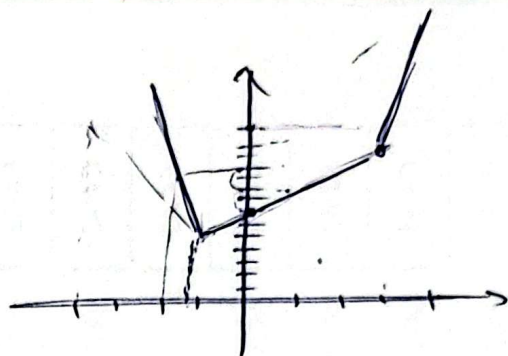
$$\frac{nr+a}{\sqrt{nr+a}} + \frac{1}{\sqrt{nr+a}}$$

9) $y = |x - r| + |x_n + c|$



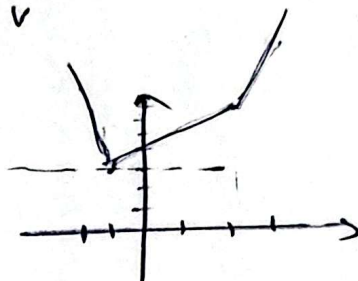
$x = c_n - c$
 $-c_{n-1}$

$x = c_n + c$
 $c_n + c$



10) $y = |x - r| + |x_n + r|$

$[c, +\infty)$



$y = |x - r| + |x_n + 1|$



$x = c_n + 1$
 $-c_{n+1}$

$x = c_n - 1 + c_n + 1$

$x = c_n + 1$
 $-c_n + c$

$\min \Rightarrow -1 + c = r$

$R_f = [r, +\infty)$

