

الف) $x^3 - 3x^2 + 3x - 1 = y - 1$
 $(x-1)^3 = y-1 \Rightarrow x = \sqrt[3]{y-1} + 1$ } $R_f = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x} \Rightarrow yx^2 - 2yx = 1 \Rightarrow yx^2 - 2yx - 1 = 0$
 $\Rightarrow x = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y} \xrightarrow{\Delta \geq 0} \begin{cases} 4y^2 + 4y \geq 0 \\ 4y(y+1) \geq 0 \end{cases}$
 $R_f = (-\infty, -1] \cup (0, +\infty)$

الف) $y = x^2 - 4x + 10$ $\begin{cases} x_{\min} = \frac{4}{2} = 2 \\ y_{\min} = 6 \end{cases}$ $R_f = [6, +\infty)$

ب) $y = -x^2 + 4x + 3$ $\begin{cases} x_{\max} = 2 \\ y_{\max} = 11 \end{cases}$ $R_f = (-\infty, 11]$

ج) $y = \sqrt{x^2 - 4x - 3}$ $\begin{cases} x_{\min} = 2 \\ y_{\min} = -1 \end{cases}$ $R_f = \sqrt{[-1, +\infty)} = [0, +\infty)$

د) $y = \sqrt{4x - x^2}$ $\begin{cases} x_{\max} = 2 \\ y_{\max} = 2 \end{cases}$ $R_f = \sqrt{[-\infty, 4]} = [0, 2]$

الف) $R_f = \mathbb{R}$

ب) $R_f = \mathbb{R}$

ج) $R_f = [0, +\infty)$

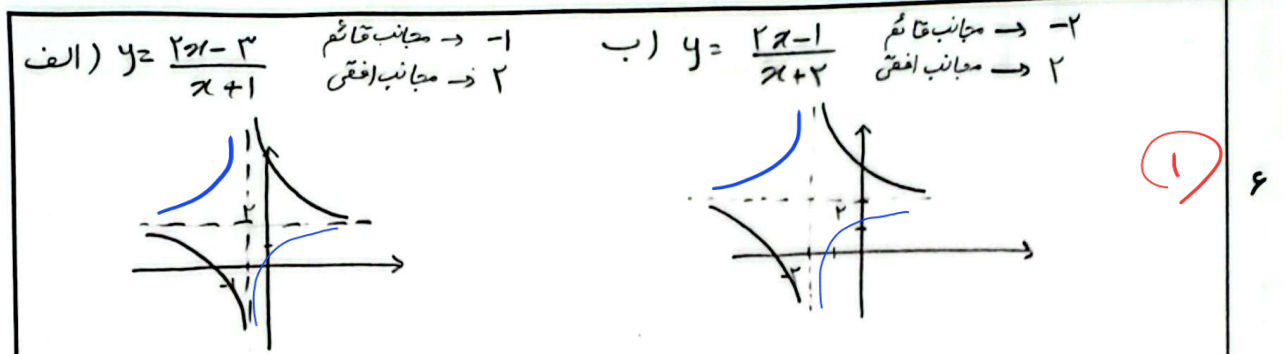
د) $R_f = \mathbb{R} \cup \{+\infty\}$

الف) $y = \frac{3x+1}{x-2}$ $R_f = \mathbb{R} - \left\{\frac{3}{1}\right\} = \mathbb{R} - \{3\}$

ب) $y = \frac{2x+1}{x+1}$ $R_f = \mathbb{R} - \{2\}$

الف) $y = \sqrt{\frac{2x+4}{x+1}}$ $R_f = \sqrt{\mathbb{R} - \{2\}} = [0, +\infty) - \{2\}$

ب) $y = \sqrt{\frac{3x+1}{2-x}}$ $R_f = \sqrt{\mathbb{R} - \{-3\}} = [0, +\infty)$



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الف) $R_f = (-\infty, -2] \cup [2, +\infty)$

ب) $y = x^3 + \frac{1}{x^3} \rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$

ج) $y = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}} \rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$

د) $R_f = [2, +\infty)$

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الف) $y = x^2 + \frac{1}{x^2+3} + 3 - 3 = x^2 + 3 + \frac{1}{x^2+3} - 3$

$3 + \frac{1}{3} - 3 = \frac{1}{3} \Rightarrow R_f = [\frac{1}{3}, +\infty)$

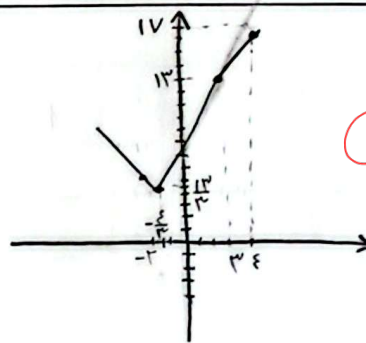
ب) $y = \frac{x^2+4+1}{\sqrt{x^2+4}} = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}} \rightarrow R_f = [2, +\infty)$

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$y = |x-3| + |3x+4|$

$f(x) = \begin{cases} x-3+3x+4 & x \geq 3 \\ 3-x+3x+4 & -\frac{4}{3} < x < 3 \\ 3-x-3x-4 & x \leq -\frac{4}{3} \end{cases}$

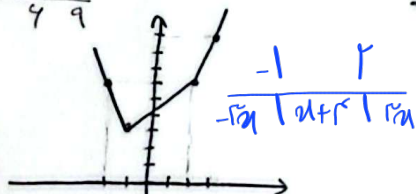
$f(x) = \begin{cases} 4x+1 & x \geq 3 \\ 2x+7 & -\frac{4}{3} < x < 3 \\ -4x-1 & x \leq -\frac{4}{3} \end{cases}$



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الف) $y = |x-2| + |2x+1|$

$\begin{matrix} -2 & -1 & 2 & 3 \\ 4 & 3 & 4 & 9 \end{matrix}$



$R_f = [3, +\infty)$

ب) $y = |2x-2| - |x+1|$

$\begin{matrix} -2 & -1 & 1 & 2 \\ 0 & 1 & -2 & -1 \end{matrix}$

$R_f = [-2, +\infty)$

$\begin{matrix} -1 & 1 \\ -2x+2 & -x-1 \end{matrix}$



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$\sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}} \xrightarrow{u^2 = 0} \frac{2}{u} \quad R = [\frac{2}{\sqrt{4}}, +\infty)$

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