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Subject

$$f(x) = \left(\frac{1}{x}\right)^{x-2} \quad y = \sqrt{x^3 f(x+1)}$$

$$f(x+1) = \frac{1}{x+1}^{(x+1)-2} = \left(\frac{1}{x+1}\right)^{x-1}$$

$$y = \sqrt{x^3 \times \left(\frac{1}{x}\right)^{x-1}}$$

$$\rightarrow x=1$$

$$x^3 \left(\frac{1}{x}\right)^{x-1} \Big|_{x=0} \rightarrow \left(\frac{1}{x}\right)^{x-1} \Big|_{x=0}$$

$$x^3 \Big|_{x=0} \rightarrow x \Big|_{x=0} \rightarrow D_f = [0, +\infty) \quad D_f = [0, 1]$$

$$f(x) = \sqrt{x+1} \sqrt{x+2}$$

$$f(-x) = \sqrt{-x+1} \sqrt{-x+2}$$

$$-x+2 \geq 0 \rightarrow x \leq 2$$

$$f(-x) = \sqrt{-x+1} \sqrt{-x+2} = \sqrt{-2x+2}$$

$$-2x+2 \geq 0 \rightarrow x \leq 1$$

$$-x+2 < 0 \rightarrow x > 2 \rightarrow f(x) = \sqrt{-x+(x-2)} = \sqrt{-2}$$

$$D_f = (-\infty, 1]$$

$$y = \sqrt{\frac{x-1}{f(x)}} \rightarrow f(x) \neq 0 \rightarrow \frac{x-1}{f(x)} \geq 0$$

$$x = x > 1, -\infty, -1$$

$$(-\infty, -1) \rightarrow f(x) > 0 \quad \times$$

$$(-1, 1) \rightarrow f(x) < 0 \quad \checkmark$$

$$(1, \infty) \rightarrow f(x) > 0 \quad \checkmark$$

$$D_f = (-1, 1] \cup (1, \infty)$$

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$$y = \frac{1}{x^2 + ax + b}$$

(۴)

$$9x^2 - 12x^2 - 4x - 1 = 2x^2 - 4x - 1$$

$$\frac{1}{x} = \frac{c}{a} = \frac{-r}{b}$$

$$(1) \frac{1}{x} = \frac{c}{a} \rightarrow 1a = -1r \rightarrow a = -r$$

$$(2) \frac{1}{x} = \frac{-r}{b} \rightarrow 1b = -r \rightarrow b = -r$$

$$a + b = -r + (-r) = -1r - r = -\frac{1r}{1} - \frac{r}{1} = -\frac{2r}{1}$$

$$\sqrt{-(a+b)} = \sqrt{-(-\frac{2r}{1})} = \sqrt{\frac{2r}{1}}$$

$$\sqrt{\frac{2r}{1}} = \frac{\sqrt{2r}}{\sqrt{1}} \times \frac{\sqrt{1}}{\sqrt{1}} = \frac{\sqrt{2r}}{1}$$

$$x^2 + ax + b = x^2 - 1x - r$$

$$a = -1, b = -r$$

$$\sqrt{-(a+b)} = 1$$

$$f(x) = x^2 + x$$

$$y = \sqrt{f(x)f(\frac{1}{x})}$$

(۵)

$$f(x) - f(\frac{1}{x}) = (x^2 + x) - ((\frac{1}{x})^2 + \frac{1}{x}) = (x^2 - \frac{1}{x^2}) + (x - \frac{1}{x})$$

$$x^2 - \frac{1}{x^2} = (x - \frac{1}{x})(x + \frac{1}{x} + \frac{1}{x^2})$$

(۵)

$$(x - \frac{1}{x})(x^2 + \frac{1}{x} + \frac{1}{x^2}) + (x - \frac{1}{x}) = (x - \frac{1}{x})(x^2 + \frac{1}{x} + \frac{1}{x^2} + 1)$$

$$x^2 - \frac{1}{x^2} \geq 0 \rightarrow \frac{(x-1)(x+1)}{x} \geq 0$$

$$D F = [-1, 0) \cup [1, \infty)$$

$$f(x^3 + x) = 4x - 1 \quad f(1) = f(x^3) = 4$$

(2)

$$x^3 + x = 1$$

(1)

$$x = 1 \rightarrow x^3 + x = 1 + 1 = 2$$

$$f(1) = f(x) = 4 - 1 = 3$$

$$x^3 + x = 2 \rightarrow x = 1 \rightarrow 1^3 + 1 = 2$$

$$f(x) = f(1) = 4 - 1 = 3 \rightarrow f(1) + f(x) = 4 - 1 = 3$$

$$g(x) = x^3 + 4x^2 + 4x \quad t = \sqrt[3]{4} - x \quad (V)$$

$$g(t) = (\sqrt[3]{4} - x)^3 + 4(\sqrt[3]{4} - x)^2 + 4(\sqrt[3]{4} - x)$$

$$= (4 - 4\sqrt[3]{4}x + 4x^2) + 4(4 - 4\sqrt[3]{4}x + x^2) - 4 + 4\sqrt[3]{4} - 4x$$

$$= 4(\sqrt[3]{4} - x)^2 + 4((\sqrt[3]{4})^2 - 4\sqrt[3]{4}x + 4)$$

$$= 4\sqrt[3]{4} - 2\sqrt[3]{4}x + 4$$

$$4\sqrt[3]{4}(\sqrt[3]{4} - x) = 4\sqrt[3]{4}\sqrt[3]{4} - 4\sqrt[3]{4}x = 4 - 4\sqrt[3]{4}x$$

$$g(-t) = (-4 + 4\sqrt[3]{4} - 4\sqrt[3]{4}x) + (4\sqrt[3]{4} - 2\sqrt[3]{4}x + 4)$$

$$= (4\sqrt[3]{4} - 4\sqrt[3]{4}x) = -4x$$

$$f(x) = (\sqrt[3]{4} + x)^3 - 4(\sqrt[3]{4} + x)^2 + 12(\sqrt[3]{4} + x)$$

$$f(x) = (4 + 4\sqrt[3]{4}x + 4x^2) + (-4\sqrt[3]{4} - 4\sqrt[3]{4}x + 4) + 12\sqrt[3]{4} + 12x$$

$$10 - 4\sqrt[3]{4} + 4\sqrt[3]{4} = 10$$

$$\frac{g(x)}{f(x)} = -\frac{4}{10} = -\frac{2}{5}$$

$$f(x) = \sqrt{x + \epsilon \sqrt{x - \epsilon}}$$

(A)

$$f(x) + g(x) = \sqrt{x - \epsilon} + \epsilon + \sqrt{x - 1}$$

$$f(n) = \sqrt{(\sqrt{n - \epsilon} + 1)^2} = \sqrt{n - \epsilon} + 1$$

$$g(n) = \sqrt{(\sqrt{n - \epsilon} - 1)^2} = |\sqrt{n - \epsilon} - 1|$$

$$x \geq 1 \rightarrow \sqrt{n - \epsilon} = a + b\sqrt{n + c} \checkmark$$

$$\frac{a+b}{c} = \frac{-1}{1}$$

$$1 \leq n \leq 1 \rightarrow a, b\sqrt{n + c},$$

(A)

$$f(x) = \frac{x+1}{(x-1)(x-2)}$$

(10)

$$x=1 \rightarrow \text{خرج}$$

$$D_f = \mathbb{R} - \{1, 2\}$$

$$x=2 \rightarrow \frac{2+1}{(2-1)(2-2)} = \frac{3}{1(-1)}$$

$$D_f \cap D_g = \{1, 2\}$$

$$\frac{g(2)}{f(2)} = \frac{1}{-1} = -1$$

$$\frac{g}{f} = \left\{ \left(1, \frac{1}{1}\right), \left(2, 1\right) \right\}$$

$$x=1 \rightarrow f(x) = 0$$

$$x=2 \Rightarrow f(2) = \frac{2+1}{(2-1)(2-2)} = \frac{3}{1(-1)} = -3$$

$$\frac{g(2)}{f(2)} = \frac{1}{-3} = -\frac{1}{3}$$



$$f(x) = \{(-1, 4), (1, 2), (3, -1), (2, 4)\} \quad (1.)$$

$$g(x) = \{(-2, 3), (1, -1), (3, 2), (-1, 0)\}$$

الف) $f \circ g$ - 1, 1, 3, 4

$$2x - 1 \rightarrow x - \frac{1}{2}$$

$$2x - 1 \rightarrow x - \frac{1}{2}$$

$$2x - 3 \rightarrow x - \frac{5}{2}$$

$$2x - 5 \rightarrow x - 2$$

$$f(g(x)) = \left\{ \left(-\frac{1}{2}, 4\right), \left(\frac{1}{2}, 2\right), \left(\frac{3}{2}, -1\right), (2, 4) \right\}$$

ب) $f(x^2)$

$$x^2 - 1 \rightarrow x \pm 1 \rightarrow f(1/2)$$

$$x^2 - 3 \rightarrow x \pm \sqrt{3} \rightarrow f(3) = 1$$

$$x^2 - 5 \rightarrow x \pm \sqrt{5} \rightarrow f(5) = 2$$

$$f(x^2) = \{(-2, 4), (-\sqrt{3}, -1), (-1, 2), (1, 2), (\sqrt{5}, -1), (2, 4)\}$$

ج) $2g(x) + 1$

$$g(-2) = 2 \times 9 + 1 = 19$$

$$g(1) = (-1)^2 = 1 \rightarrow 2 \times 1 + 1 = 3$$

$$g(3) = 3^2 = 9 \rightarrow 2 \times 9 + 1 = 19$$

$$g(-1) = 0^2 = 0 \rightarrow 2 \times 0 + 1 = 1$$

$$2g(x) + 1 = \{(-2, 19), (1, 3), (3, 19), (-1, 1)\}$$

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$$f(x) = \sqrt{x-1} \sqrt{x-1}$$

(1)

$$g(x) = \sqrt{x-1} \sqrt{x-1}$$

$$(f(x) + g(x)) = a + b \sqrt{x+c}$$

$$(x+1 \sqrt{x-1}) + (x-1 \sqrt{x-1}) = 2 \sqrt{(x+1 \sqrt{x-1})(x-1 \sqrt{x-1})}$$

$$2x + 2(x+1) = 2x + 14$$

$$g(x) + g(x) = \sqrt{x+14} = 2 \sqrt{x+7}$$

$$a=0 \quad b=2 \quad c=7 \rightarrow \frac{x+b}{c} = \frac{x+7}{7} = \frac{1}{7}$$