حل تمرین ۵

① $f(x)=\left(\frac{1}{2}\right)^{x-1}$ $\quad g:\sqrt{x^2 f(x+1)}$ $\quad D_g$ ؟

$f(x)\geq 0 \rightarrow \left(\frac{1}{2}\right)^{x-1}\geq 1 \Rightarrow x-1\leq 0 \Rightarrow x\leq 1$

$f(x+1)\geq 0 \rightarrow \left(\frac{1}{2}\right)^{x-1}\geq 0 \Rightarrow x-1\leq 0 \Rightarrow x\leq 1$ ★

$x^2\geq 0 \Rightarrow x\geq 0$ ★ $\quad x^2 f(x+1)\geq 0 \xrightarrow{★,★} 0\leq x\leq 1$ $\quad D_f=[0,1]$

② $f(x)=\sqrt{x+|x+2|}$ $\quad f(x)$ ؟ $\quad D_f=(-\infty,1]$

$f(-x)\overset{x:=-x}{\Rightarrow}\sqrt{-x+|-x+2|}$ $\quad -x+|-x+2|\geq 0$

✓	✗
$-x-x+2=-2x+2$	$-x+x-2=-2\Rightarrow -2x+2\geq 0$
	$-2x\geq -2$
	$x\leq 1$

③ $\sqrt{\frac{x-1}{f(x)}}$ $\quad \frac{x-1}{f(x)}\geq 0$

x	$-\infty$		1		2		3		$+\infty$
$x-1$	$-$	$-$	0	$+$	$+$	$+$	$+$	$+$	$+$
$f(x)$	$+$	0	$-$	$-$	0	$+$	0	$+$	$+$
	$-$	$\nexists$	$+$	0	$-$	$\nexists$	$+$	0	$+$

$D_f=(-\infty,1]\cup(2,3)$

۴

$$\frac{1}{9x^2-7x^2-4x-3} = y_1$$

$$y_2 = \frac{1}{3x^2+ax+b}$$

$D_1 = D_2$ ← عبارت را به عبارتی ضرب می‌کنیم که $3x^2$ با $9x^2$ کرده و ریشه نداشته باشد.

$\Delta > 0$ ، $c = d = 1$

$$(3x^2+9x+b)(3x^2+cx+d)$$

$$9x^4+3(a+c)x^3+(ac+3(b+d))x^2+(ad+bc)x+bd$$

$$a+c=0 \rightarrow a=-c \qquad \underbrace{ac}_{-c^2}+3b+3d=-7$$

$$\begin{cases} -c^2+3(b+d)=-7 \\ c(b-d)=-4 \end{cases} \qquad ad+bc=-6$$

$$-cd+bc=-4 \quad c(b-d)=-4$$

$c=1 \rightarrow -1+3(b+d)=-7 \rightarrow 3(b+d)=-9$

$$b+d=-3$$

$$\begin{cases} b+d=-3 \\ b-d=-4 \end{cases} \Rightarrow 2b=-9 \rightarrow b=-3$$

$$-3+d=-1 \Rightarrow d=1 \quad c=-a \Rightarrow a=-1$$

$$\sqrt{-(a+b)} = \sqrt{-(\frac{-3}{-4}-1)} = \sqrt{4} = 2$$

(۷)

$f(x) = x^3 - 6x^2 + 12x - 8 + 8$

$= (x-2)^3 + 8$

$g(x) = x^3 + 9x^2 + 27x + 27 - 27 = (x+3)^3 - 27$

$$\frac{g(\sqrt[3]{y-8}+3)}{f(\sqrt[3]{x-4}+2)} = \frac{y-27}{x+8} = \frac{-20}{10} = -2$$

(۶)

$f(x^2+x) = 2x^2 - 1$

$f(2) \to x^2 + x = 2 \to x^2 + x - 2 = 0 \to (x-1)(x^2+x+2)$

$x = 1$

$\Rightarrow f(2) = 1$

$f(x^2+x) \xrightarrow{x=2} f(6) \xrightarrow{x=1} f(6) = 2(2)^2 - 1 = 7$

$f(2) + f(6) = 7 + 1 = 8$

(۵)

$\sqrt{f(x) - f(\frac{4}{x})} \qquad \sqrt{x^2 + x - \frac{4x}{x^2} - \frac{4}{x}}$

$$\frac{x^4 + x^3 - 4x^2 - 4x}{x} \geq 0$$

$y = x^4 + x^3 - 4x^2 - 4x \geq 0$

$x > 0$

$t^2 + 5t + 4 = 0$

$x = -2 \to y = 0$

$x = -1 \to y = 0$

$\to$ تشکیل $x(x^2-2)$

$(x^2-2)(x^2+3x+4)$

$\Delta < 0$

	-2	0	2	
	$+$	$-$	$+$	

$Df = [-2, 0) \cup [2, +\infty)$

(۱۰)

الف) $f(2x) = \{(\frac{1}{2}, 2)(\frac{3}{2}, -1)(2, 2)(-\frac{1}{2}, 9)\}$

ب) $f(x^2) = \{(1, 2)(\sqrt{3}, -1)(2, 2)(-1, 2)(-\sqrt{3}, -1)$
$(-2, 2)$

ج) $2g^2(x)+1 \Rightarrow \{(-2, 19)(1, 3)(3, 9)(-1, 1)\}$

د) $\frac{2f}{g} \Rightarrow \{(1, \frac{-4}{-4})(3, -1)~~(-1, \frac{4}{0})\}$

(۹)

$f(x) = \frac{x+2}{x^2-4x+3}$ ~~~ مشترک ها $f(x) = \{(2, -2)(1, \ldots$

$f(x)$
$\{(2, -4)~~(1, \frac{3}{0})~~(3, \frac{5}{0})~~(0, \frac{2}{3})\}$

$\frac{g}{f} = \{(2, \frac{1}{-4})(0, \frac{1}{\frac{2}{3}} = 9)\}$

(۸)

$f(x) = \sqrt{x+4\sqrt{x-4}} = \sqrt{(\sqrt{x-4}+2)^2} = |\sqrt{x-4}+2|$

$g(x) = \sqrt{x-4\sqrt{x-4}} = \sqrt{(\sqrt{x-4}-2)^2} = |\sqrt{x-4}-2|$ ~~~ هر چقدر x بزرگتر از ۴ باشد قدر مطلق منفی نیست

$f(x)+g(x) = \sqrt{x-4} ~~+2~~ + \sqrt{x-4} ~~-2~~ = 2\sqrt{x-4}$

$\Rightarrow a=0,~ b=2,~ C=-4 \quad \frac{a+b}{C} = \frac{2}{-4} = -\frac{1}{2}$