

نام و نام خانوادگی شماره کلاس **پاسخنامه تشریحی تکلیف شماره ۱۴، ۸**

$$\rightarrow D_f = (-\infty, 0) \cup \left[\frac{1}{2}, 1\right)$$

الف) $f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1}$

$x \neq 0$
 $x \neq 1$
 $x = \frac{1}{y}$

$\frac{x-1}{x} = \frac{\frac{1}{y}-1}{\frac{1}{y}} = 1-y$
 $\frac{x}{x-1} = \frac{\frac{1}{y}}{\frac{1}{y}-1} = \frac{1}{1-y}$

$f(x) = \sqrt{1-y} - \frac{1}{1-y}$

$1-y \geq 0 \Rightarrow y \leq 1$
 $1-y \neq 0 \Rightarrow y \neq 1$

$D_f = \left(\frac{1}{2}, 1\right) \cup \left[\frac{1}{2}, 1\right)$

$f(x) = \frac{1}{x+1} - \frac{x}{x}$

$\frac{1}{x-1} + \frac{1}{x+1}$

$x \neq 0$
 $x \neq -1$
 $x \neq 1$
 $x \neq -2$

$D_f = \mathbb{R} - \{-2, -1, 0, 1\}$

$\frac{x}{x-1} + \frac{1}{x+1} \neq 0 \Rightarrow x = -\frac{2}{x}$

ب) $f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^x - 4\right)(x^2 - 4x)}$

$(x^2 - 4x) \geq 0$
 $x^2 - 4x = 0 \Rightarrow x = 0$
 $x^2 - 4x = 0 \Rightarrow x = 4$

$(\frac{1}{x})^x - 4 \geq 0$
 $(\frac{1}{x})^x \geq 4$
 $x < 0$
 $x > 4$

$D_f = (-\infty, -2] \cup \left[\frac{4}{3}, +\infty\right)$

$\sqrt{x-1} + \sqrt{y+1} = 1$

$x-1 \geq 0 \Rightarrow x \geq 1$
 $y+1 \geq 0 \Rightarrow y \geq -1$

$D_f = \{(x, y) \mid x \geq 1, y \geq -1\}$

$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$

$D_f = \mathbb{S}$

$(-\infty, -1) \cup (2, +\infty)$

$x^2 - x - 2 > 0$
 $(x-2)(x+1) > 0$
 $x < -1$
 $x > 2$

$x^2 - 1 > 0$
 $x^2 > 1$
 $x < -1$
 $x > 1$

$f(x) = \sqrt{3+ax-x^2} \Rightarrow [-2, b] = a+b=5$

$-x^2+ax+3 \geq 0$
 $x^2-ax-3 \leq 0$
 $x^2-ax-3=0$
 $x = \frac{a \pm \sqrt{a^2+12}}{2}$

$-f = a - \sqrt{a^2+12}$
 $\sqrt{a^2+12} \Rightarrow a^2+12 \geq 0 \Rightarrow a^2+12 = a^2+12+12$
 $a \geq -2$
 $a = -\frac{1}{2}$

$b = \frac{3}{2}$

$f(x) = \begin{cases} 3x-2 & ; x \geq 1 \\ 2x+3 & ; x < 1 \end{cases}$

$D_g = \mathbb{S}$

$g(x) = \sqrt{f(x) - x}$

$f(x) - x \geq 0$

۱) $3x-2-x \geq 0 \Rightarrow x \geq 1$

۲) $2x+3-x \geq 0 \Rightarrow x \geq -3 \Rightarrow x < 1 \Rightarrow -3 \leq x < 1 \Rightarrow [-3, 1) \cup [1, +\infty)$

$$f(x) = \begin{cases} (a+1)(x+1) & ; x > 1 \\ pa+px & ; x \leq 1 \end{cases} \quad \text{if } f(x) = f(-x) + a \quad a=5$$

$$(a+1)(1) = 1a+1 \quad \text{if } (1a+1) = 1a-5+a$$

$$1a+1(-1) = 1a-5 \quad 1a+1(-1) = 1a-5$$

$$1a = -1 \Rightarrow a = -1$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 1$$

$$f(1-\sqrt{x}) + f(1+\sqrt{x}) = 5$$

$$\left(\sqrt{1-\sqrt{x}} + \frac{1}{\sqrt{1-\sqrt{x}}} + 1 \right) + \left(\sqrt{1+\sqrt{x}} + \frac{1}{\sqrt{1+\sqrt{x}}} + 1 \right) =$$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$5 + 2ab$$

$$ab = \sqrt{(1-\sqrt{x})(1+\sqrt{x})} = 1$$

$$a+b = \sqrt{4}$$

$$\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{\sqrt{4}}{1} = \sqrt{4}$$

$$\begin{cases} f(x) - pf(-x) = cx^2 - x \\ pf(-x) - pf(x) = cx^2 + x \end{cases}$$

$$f(x) = ?$$

$$f(x) = -cx^2 - \frac{1}{2}x$$

$$\begin{cases} 4f(x) - pf(-x) = 12x^2 - 2x \\ pf(-x) - pf(x) = 4x^2 + 2x \\ -2f(x) = 10x^2 + 2x \end{cases}$$

$$f(-x) = \frac{10x^2 - 2x}{-2}$$

$$f(x) = \frac{10x^2 - 2x}{2} = \frac{x(10x-1)}{2}$$

$$(x+1)f(x) - px f(x+1) = cx^2 - mx + pm - 1 \quad f(0) = ?$$

$$pf(0) - 0 = 0 - 0 + pm - 1$$

$$x = -1 \Rightarrow f(0) = \frac{18 + 2m}{4}$$

$$f(0) = \frac{pm-1}{p}$$

$$\frac{pm-1}{p} = \frac{18+2m}{4} \Rightarrow m = \frac{4}{p}$$

$$f(0) = \frac{18}{p}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{px^2 - 12x + 3}{x} \quad (x \in \mathbb{R} \setminus \{0\})$$

$$f(-1) = ?$$

$$f(-1) + f(-1) = \frac{3+12+3}{-1} \Rightarrow pf(-1) = -18$$

$$f(-1) = -9$$

$$x-1 \geq 0 \rightarrow x \geq 1$$

$$(\leftarrow)^P$$

$$\underbrace{x-1}_{\geq 0} \leq \underbrace{\sqrt{y+1}}_{+} \rightarrow x \leq \wedge P \quad D_f = [1, \wedge P]$$