

اینجا ترمیم لازم - باز هم 8 - تطبیق 4

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} = \sqrt{\frac{(x-1)^2 - x^2}{x(x-1)}} \Rightarrow f(x) = \sqrt{\frac{1-2x}{x(x-1)}}$ سوال 1

$\begin{matrix} 0 & \frac{1}{2} & 1 \\ + & | & - \\ 0 & 0 & 0 \end{matrix}$

$D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x-1} + \frac{1}{x+2}} = \frac{\frac{x - (x+1)}{(x+1)x} \neq 0 \rightarrow x \neq 0, -1}{\frac{1}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow x \neq -\frac{2}{3}, \frac{1}{2}}$

$\frac{1}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow x \neq 1, -2$

$D_f = \mathbb{R} - \{-2, -\frac{2}{3}, -1, 0, \frac{1}{2}\}$

الف) $f(x) = \sqrt{((\frac{1}{x})^2 - 9)(x^2 - 1)} \rightarrow (x^2 - 1)(\frac{1}{x^2} - 1) \rightarrow \frac{-x=1}{x=-1} \quad \frac{x=0}{x=\frac{0}{1}} \quad \begin{matrix} -1 & \frac{0}{1} \\ + & | & - \\ 0 & 0 & 0 \end{matrix}$ سوال 2

$D_f = \mathbb{R} - (-2, \frac{0}{1})$

ب) $\sqrt{x-1} + \sqrt{y+1} = 2 \rightarrow \sqrt{y+1} = 2 - \sqrt{x-1} \Rightarrow 2 \geq \sqrt{x-1} \rightarrow 1 \geq x-1 \rightarrow 1 \geq x$

$\sqrt{x-1} \geq 0 \rightarrow x \geq 1$

$D_f = [1, 1]$

$f(x) = \frac{\log_2(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} = \frac{\log_2 \frac{(x-2)(x+1) > 0 \rightarrow \frac{-1}{0} \frac{2}{0} +}{\sqrt{(x-1)(x+1)} + 1}$ سوال 3

$\begin{matrix} -1 & 2 \\ + & | & - \\ 0 & 0 & 0 \end{matrix}$

$D_f = (-\infty, -1) \cup (2, +\infty)$

$f(x) = \sqrt{x+ax-x^2} \geq 0 \quad D_f = [r, b] \rightarrow \frac{-r}{0} \frac{b}{0} \quad x=-2 \quad x-r-a-x^2=0 \rightarrow ra=-1 \rightarrow a=-\frac{1}{r}$ سوال 4

$\begin{matrix} -r & b \\ - & | & - \\ 0 & 0 & 0 \end{matrix}$

$a+b = \frac{r}{r} + (-\frac{1}{r}) = 1$

$f(x) = \begin{cases} x-2 & : x > 1 \\ x+2 & : x < 1 \end{cases} \rightarrow \begin{cases} x-2 \geq 0 \rightarrow x \geq 2 \\ x+2 \geq 0 \rightarrow x \geq -2 \end{cases}$ سوال 5

$\Rightarrow D_g = \textcircled{1} \cup \textcircled{2} = [-2, +\infty)$

$g(x) = \sqrt{f(x)-x} \geq 0 \rightarrow f(x) \geq x$

$f(x) = \begin{cases} (a+1)(x+2) & : x > 1 \\ ra+rx & : x \leq 1 \end{cases} \rightarrow (a+1)x+2a+2$

$r f(a) = f(-r) + a$

$r(2a+2a+2a+2) = ra - r + a \rightarrow r(4a+2) = ra - r$

$\rightarrow 10a = -1 \rightarrow a = \frac{-1}{10}$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$$

$$f(\underbrace{x-\sqrt{x}}_{a, \frac{1}{a}}) + f(\underbrace{x+\sqrt{x}}_{\frac{1}{a}, a}) = \underbrace{\sqrt{x-\sqrt{x}}}_a + \underbrace{\frac{1}{\sqrt{x-\sqrt{x}}}}_{\frac{1}{a}} + 2 + \underbrace{\sqrt{x+\sqrt{x}}}_{\frac{1}{a}} + \underbrace{\frac{1}{\sqrt{x+\sqrt{x}}}}_a + 2$$

$$\Rightarrow 2\left(\sqrt{a+\frac{1}{a}}\right)^2 = 2\left(\sqrt{x-\sqrt{x}} + \frac{1}{\sqrt{x-\sqrt{x}}}\right)^2 = 2\sqrt{x-\sqrt{x}} + \frac{2}{\sqrt{x-\sqrt{x}}} + 2 = 2\sqrt{x-\sqrt{x}} + 2\sqrt{x+\sqrt{x}} + 4$$

$$2f(x) - 2f(-x) = 2x^2 - x \quad \begin{cases} 2f(x) - 2f(-x) = 2x^2 - x \\ 2f(x) - 2f(x) = 2x^2 + x \end{cases}$$

$$\begin{cases} 4f(x) - 4f(-x) = 4x^2 - 2x \\ -4f(x) + 4f(-x) = 2x^2 + 2x \end{cases}$$

$$-8f(x) = 20x^2 + x$$

$$\rightarrow f(x) = \frac{-20x^2 - x}{8} = \boxed{-\frac{5x^2}{2} - \frac{x}{8}}$$

$$(x+2)f(x) - 2xf(x+2) = 2x^2 - mx + 2m - 1 \quad \xrightarrow{x=0} \quad (2)f(0) = 2m - 1$$

$$\xrightarrow{x=-2} \quad 4f(0) = 14 + 2m + 2m - 1$$

$$\begin{cases} 4f(0) = 9m - 2 \\ 4f(0) = 13 + 4m \end{cases}$$

$$\Rightarrow 4f(0) = 9m - 2 = 4m + 13$$

$$\rightarrow m = \frac{15}{5} = 3$$

$$\rightarrow 4f(0) = 27 - 2 = 25$$

$$\rightarrow \boxed{f(0) = \frac{25}{4}}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{2x^2 - 12x + 2}{x} \quad \xrightarrow{x=-1} \quad 2f(-1) = \frac{14}{-1} \rightarrow \boxed{f(-1) = -7}$$