آیناز کریم زاده - یازدهم B - تکلیف ۴

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سوال ۱)

الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}=\sqrt{\frac{(x-1)^2-x^2}{x(x-1)}} \to ((x-1)-x)((x-1)+x)$

$\Rightarrow f(x)=\sqrt{\frac{1-2x}{x(x-1)}}$

$0 \quad \frac{1}{2} \quad 1$: $+ \,|\, - \,|\, + \,|\, -$ (ت ن ، 0 ، ت ن)

$D_f=(-\infty,0)\cup[\frac{1}{2},1)$

ب) $f(x)=\frac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}=\frac{\frac{x-2(x+1)}{(x+1)x}}{\frac{3(x+2)+(x-1)}{(x-1)(x+2)}}$

$(x+1)x\neq 0 \to x\neq 0,-1$

$3(x+2)+(x-1)\neq 0 \to x\neq -\frac{5}{4}$ ($4x+5$)

$(x-1)(x+2)\neq 0 \to x\neq 1,-2$

$\Rightarrow D_f=\mathbb{R}-\{-2,-\frac{5}{4},-1,0,1\}$

سوال ۲)

الف) $f(x)=\sqrt{((\frac{1}{3})^x-9)(32-4^x)} \to (3^{-x}-3^2)(2^5-2^{2x}) \to \begin{matrix}-x=2\\ x=-2\end{matrix} \quad \begin{matrix}2x=5\\ x=\frac{5}{2}\end{matrix}$

$-2 \quad \frac{5}{2}$: $+ \,|\, - \,|\, +$ (0 ، 0)

$D_f=\mathbb{R}-(-2,\frac{5}{2})$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3 \to \underbrace{\sqrt{y+1}}_{\geq 0}=\underbrace{3-\sqrt[4]{x-1}}_{\geq 0} \Rightarrow 3\geq \sqrt[4]{x-1} \to 81\geq x-1 \to 82\geq x$

$\sqrt[4]{x-1}\geq 0 \to x\geq 1$

$D_f=[1,82]$

سوال ۳)

$f(x)=\frac{\log_4^{(x^2-x-2)}}{\sqrt{x^2-1}+1}=\frac{\log_4^{(x-2)(x+1)}}{\sqrt{(x-1)(x+1)}+1}$

$(x-2)(x+1)>0 \to$ $-1 \quad 2$: $+ \,|\, - \,|\, +$ (ت ن ، ت ن) ②

↳ $-1 \quad 1$: $+ \,|\, - \,|\, +$ (0 ، 0) ①

$① \cap ② = (-\infty,-1)\cup(2,+\infty)$

سوال ۴)

$f(x)=\sqrt{3+ax-x^2}\geq 0 \quad D_f=[-2,b] \to$ $-2 \quad b$: $- \,|\, + \,|\, -$ (0 ، 0) $\xrightarrow{x=-2} 3-2a-4=0 \to 2a=-1 \to a=-\frac{1}{2}$

↳ $\sqrt{-(x^2+\frac{1}{2}x-3)}$

↳ $\alpha+\beta=-\frac{1}{2}$, $\alpha\beta=-3$ $\quad -2+\beta=-\frac{1}{2} \to \beta=\frac{3}{2} \Rightarrow b=\frac{3}{2}$

$a+b=\frac{3}{2}+(-\frac{1}{2})=1$

سوال ۵)

$f(x)=\begin{cases}3x-2 & : x\geq 1\\ 2x+3 & : x<1\end{cases}$

$\to 3x-2\geq x \to 2x\geq 2 \to x\geq 1$ ①

$\to 2x+3\geq x \to x\geq -3 \to -3\leq x<1$ ②

$g(x)=\sqrt{f(x)-x}\geq 0 \to f(x)\geq x$

$\Rightarrow D_g=①\cup②=[-3,+\infty)$

سوال ۶)

$f(x)=\begin{cases}(a+1)(x+2) & : x>1\\ 3a+2x & : x\leq 1\end{cases}$ $\quad (a+1)x+2a+2$

$2f(5)=f(-2)+a$

$2(5a+2a+5+2)=3a-4+a \to 2(7a+7)=4a-4$

$\to 10a=-18 \to \boxed{a=\frac{-18}{10}}$ $\quad -1.8$

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سوال ۷)

$$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2$$

$$\underbrace{f(2-\sqrt{3})+f(2+\sqrt{3})}_{a,\ \frac{1}{a}}=\underbrace{\sqrt{2-\sqrt{3}}}_{a}+\underbrace{\frac{1}{\sqrt{2-\sqrt{3}}}}_{\frac{1}{a}}+2+\underbrace{\sqrt{2+\sqrt{3}}}_{\frac{1}{a}}+\underbrace{\frac{1}{\sqrt{2+\sqrt{3}}}}_{a}+2$$

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$$\Rightarrow 2\left(\sqrt{a}+\frac{1}{\sqrt{a}}\right)^2=2\left(\sqrt[4]{2-\sqrt{3}}+\frac{1}{\sqrt[4]{2-\sqrt{3}}}\right)^2=2\sqrt{2-\sqrt{3}}+\frac{2}{\sqrt{2-\sqrt{3}}}+4=2\sqrt{2-\sqrt{3}}+2\sqrt{2+\sqrt{3}}+4$$

$$2\Big(\underbrace{\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}}_{A}\Big)=2\sqrt{6}+4$$

$$A^2=6$$

سوال ۸)

$$2f(x)-3f(-x)=4x^2-x$$

$$\rightarrow 2f(x)-3f(-x)=4x^2-x$$

$$\rightarrow 2f(-x)-3f(x)=4x^2+x$$

$$\begin{cases}4f(x)-6f(-x)=8x^2-2x\\-9f(x)+6f(-x)=12x^2+3x\end{cases}$$

$$-5f(x)=20x^2+x$$

$$\rightarrow f(x)=\frac{-20x^2-x}{5}=\boxed{-4x^2-\frac{x}{5}}$$

سوال ۹)

$$(x+2)f(x)-3xf(x+2)=4x^2-mx+3m-1\xrightarrow{x=0}(2)f(0)=3m-1$$

$$\xrightarrow{x=-2}6f(0)=16+2m+3m-1$$

$$\begin{cases}6f(0)=9m-3\\6f(0)=15+5m\end{cases}$$

$$\Rightarrow 6f(0)=9m-3=5m+15$$

$$\rightarrow m=\frac{18}{4}=4.5$$

$$\rightarrow 6f(0)=37.5$$

$$\rightarrow \boxed{f(0)=6.25}$$

سوال ۱۰)

$$f(x)+f\left(\frac{1}{x}\right)=\frac{3x^2-12x+3}{x}\xrightarrow{x=-1}2f(-1)=\frac{18}{-1}\rightarrow\boxed{f(-1)=-9}$$