

$$(الف) f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \geq 0 \Rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0$$

$$\Rightarrow \frac{-1x+1}{x^2-x} \geq 0 \Rightarrow \frac{-1x+1}{x(x-1)} \geq 0$$

$$\Rightarrow \begin{cases} x_1 = \frac{1}{x} \\ x_2 = 0 \\ x_3 = 1 \end{cases} \Rightarrow D_f = (-\infty, 0) \cup \left[\frac{1}{x}, 1\right)$$

$$(ب) f(x) = \frac{1}{x+1} - \frac{1}{x} \Rightarrow \begin{cases} (I): x+1 \neq 0 \Rightarrow x \neq -1 \\ (II): x \neq 0 \end{cases}$$

$$\frac{1}{x-1} + \frac{1}{x+2} \Rightarrow \begin{cases} (III): x-1 \neq 0 \Rightarrow x \neq 1 \\ (IV): x+2 \neq 0 \Rightarrow x \neq -2 \end{cases}$$

$$\frac{1(x+2) + x-1}{(x-1)(x+2)} \Rightarrow \frac{2x+1}{(x-1)(x+2)} \neq 0 \Rightarrow \begin{cases} x \neq 1 \\ x \neq -2 \end{cases}$$

$$2x+1 \neq 0 \Rightarrow x \neq -\frac{1}{2} \Rightarrow D_f = \mathbb{R} - \left\{-2, -1, -\frac{1}{2}, 0, 1\right\}$$

$$(الف) f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^x - 9\right)(32 - 4^x)} \Rightarrow \left(\left(\frac{1}{x}\right)^x - 9\right)(32 - 4^x) \geq 0$$

$$\left(\frac{1}{x}\right)^x - 9 \geq 0 \Rightarrow 3^{-x} \geq 3^2 \Rightarrow -x \geq 2 \Rightarrow x \leq -2$$

$$32 - 4^x \geq 0 \Rightarrow 4^x \leq 32 \Rightarrow 2^{2x} \leq 2^5 \Rightarrow x \leq \frac{5}{2}$$

$$\begin{cases} \left(\frac{1}{x}\right)^x - 9 < 0 \Rightarrow 3^{-x} < 9 \Rightarrow x > -2 \\ 32 - 4^x < 0 \Rightarrow 4^x > 32 \Rightarrow x > \frac{5}{2} \end{cases} \Rightarrow D_f = (-\infty, -2] \cup \left[\frac{5}{2}, +\infty\right)$$

$$(ب) \sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \begin{cases} x-1 \geq 0 \Rightarrow x \geq 1 \\ y+1 \geq 0 \Rightarrow y \geq -1 \end{cases}$$

$$D_f = [1, +\infty)$$

$$f(x) = \frac{(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \quad (I): x^2 - x - 2 > 0 \Rightarrow (x+1)(x-2) > 0 \Rightarrow \begin{array}{c|c|c|c} -1 & 2 & & \\ \hline + & - & + & + \end{array} \quad (4)$$

$$(II): x^2 - 1 > 0 \Rightarrow (x-1)(x+1) > 0 \Rightarrow \begin{array}{c|c|c|c} -1 & 1 & & \\ \hline + & - & + & + \end{array} \quad (-\infty, -1] \cup [1, +\infty)$$

$$D_f = I \cap II = (-\infty, -1) \cup (1, +\infty)$$

$$\sqrt{x^2 + ax - x^2} \quad [-1, b] \quad a+b=? \quad x^2 + ax - x^2 > 0 \quad \text{با توجه به اینکه بازه } [-1, b] \text{ را داریم}$$

$$x^2 - 2a - x^2 = 0 \quad -2a = 1 \quad \text{این عبارت یکای } x \text{ در سمت راست داریم}$$

$$a = -\frac{1}{2} \quad \text{پس } x^2 - \frac{1}{2}x - x^2 > 0 \Rightarrow 0 < \frac{1}{2} - \frac{1}{2}(1)(-1) = \frac{29}{4}$$

$$\frac{\pm 1}{2} \pm \sqrt{\frac{29}{4}} \quad \begin{cases} x_1 = 2 \rightarrow b \\ x_2 = -\frac{3}{2} \end{cases} \quad \text{پس } a+b = -\frac{1}{2} + 2 = \frac{3}{2}$$

$$f(x) = \begin{cases} x^2 - 2; & x \geq 1 \\ x^2 + 3; & x < 1 \end{cases} \quad g(x) = \sqrt{f(x)} - x \quad f(x) - x > 0 \quad f(x) > x$$

$$\begin{cases} x^2 - 2 > x \Rightarrow x^2 > x + 2 \Rightarrow x > 1 \\ x^2 + 3 > x \Rightarrow x > -3 \end{cases} \quad \text{در صورت اول } x < 1 \text{ پس باید است}$$

$$\Rightarrow -3 \leq x < 1 \quad II$$

$$D_f = I \cap II = [-3, +\infty)$$

$$f(x) = \begin{cases} (a+1)(x+2); & x > 1 \\ 2a + 2x; & x \leq 1 \end{cases} \quad f'(x) = f'(-1) + a$$

$$x = a \Rightarrow (a+1)(v) = va + v$$

$$2x(va + v) = 2a + 2x \quad x = -1 \Rightarrow 2a - 2$$

$$\Rightarrow 2a + 2x = 2a - 2 + a \Rightarrow 10a = -10 \quad a = -1$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \Rightarrow \sqrt{2+\sqrt{2}} + \frac{1}{\sqrt{2+\sqrt{2}}} + 2 \quad \text{با توجه به اینکه}$$

$$\Rightarrow \frac{\sqrt{2+\sqrt{2}}}{2} \Rightarrow \frac{\sqrt{2+\sqrt{2}}}{2} + \frac{1}{\frac{\sqrt{2+\sqrt{2}}}{2}} + 2 \Rightarrow \frac{\sqrt{2+\sqrt{2}}}{2} + \frac{\sqrt{2-\sqrt{2}}}{2} + 2$$

$$\Rightarrow \sqrt{2} + 2$$

$$\Rightarrow f(2-\sqrt{2}) \frac{\sqrt{2}-\sqrt{2}}{2} + \frac{1}{\frac{\sqrt{2}-\sqrt{2}}{2}} + 2 \Rightarrow \frac{\sqrt{2}-\sqrt{2}}{2} + \frac{2}{\sqrt{2}-\sqrt{2}} + 2$$

$$\Rightarrow \frac{\sqrt{2}-\sqrt{2}}{2} + \frac{\sqrt{2}+\sqrt{2}}{2} + 2 \Rightarrow \sqrt{2} + 2 \quad f(2+\sqrt{2}) + f(2-\sqrt{2})$$

$$\Rightarrow \boxed{2\sqrt{2} + 4}$$

$$1f(x) - 1f(-x) = 1x^1 - x \Rightarrow (-x) \Rightarrow \begin{cases} 1f(-x) - 1f(x) = 1x^1 + x & \times 1 \\ -1f(-x) + 1f(x) = 1x^1 - x & \times 1 \end{cases} \quad (1)$$

$$\Rightarrow \begin{cases} 1f(-x) - 1f(x) = 1x^1 + x \\ -1f(-x) + 1f(x) = 1x^1 - x \end{cases} \Rightarrow f(x) = \frac{-1x^1 - 1}{2}x$$

$$\frac{-2f(x) = 1x^1 - x}{-2f(x) = 1x^1 - x} \Rightarrow f(x) = \frac{-1x^1 - 1}{2}x$$

$$(x+1)f(x) - 1xf(x+1) = 1x^1 - mx + 1m - 1 \quad f(0) = ? \quad (9)$$

$$\hookrightarrow x = -1 \Rightarrow 0 = 1 + 1m + 1m - 1 \Rightarrow 2m = -1 \Rightarrow m = -\frac{1}{2}$$

$$\hookrightarrow x = 0 \Rightarrow 1f(0) = 1m - 1 \Rightarrow 1(-\frac{1}{2}) - 1 = -1. \Rightarrow f(0) = -1 \quad \text{جواب}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{1x^1 - 11x + 1}{x} \quad f(-1) = ? \quad (10)$$

$$\frac{1x^1 - 11x + 1}{x}$$

$$ax + b + \left(a\frac{1}{x} + b\right) = ax + \frac{a}{x} + 1b$$

$$\Rightarrow ax + \frac{a}{x} + 1b = 1x - 11 + \frac{1}{x} \Rightarrow a = 1, b = -11$$

$$f(x) = 1x - 11 \Rightarrow f(-1) = 1(-1) - 11 = -12 \quad \text{جواب}$$