دنیا شرفیا گروه دختران B E

۱ - الف)
$$\frac{x-1}{x}-\frac{x}{x-1}\geq 0 \rightarrow \frac{x^2-2x+1-x^2}{x(x-1)}\geq 0 \qquad x=\frac{1}{2},\ x=0,\ x=1$$

x		0		$\frac{1}{2}$		1	
$-2x+1$	$+$		$+$		$-$		$-$
$x(x-1)$	$+$		$-$		$-$		$+$
			$-$				$-$

$$D_f=(-\infty,0)\cup[\tfrac{1}{2},1)$$

ب)
$$x+1\neq 0\rightarrow x\neq -1 \qquad x\neq 0 \qquad x-1\neq 0\rightarrow x\neq 1$$
$$x+2\neq 0\rightarrow x\neq -2$$
$$\frac{3}{x-1}+\frac{1}{x+2}\neq 0\rightarrow \frac{3}{x-1}\neq\frac{-1}{x+2}\rightarrow 3x+6\neq -x+1$$
$$4x\neq -5\rightarrow x\neq -\frac{5}{4}\rightarrow D_f=R-\{0,-1,1,-2,-\tfrac{5}{4}\}$$

۲ - الف)
$$\left(\tfrac{1}{3}\right)^x-9=0\rightarrow\left(\tfrac{1}{3}\right)^x=3^2\rightarrow x=-2$$
$$32-4^x=0\rightarrow 2^5=2^{2x}\rightarrow x=\frac{5}{2}$$

x		-2		$2/5$	
$(\frac{1}{3})^x-9$	$+$		$-$		$-$
$32-4^x$	$+$		$+$		$-$
	$+$		$-$		$+$

$$D_f=(-\infty,-2]\cup[\tfrac{5}{2},+\infty)$$

ب)
$$\sqrt{y+1}=3-\sqrt[4]{x-1}\rightarrow 3-\sqrt[4]{x+1}\geq 0\rightarrow 3\geq\sqrt[4]{x+1}$$
$$\xrightarrow{(\ )^4} 81\geq x-1\rightarrow x<82 \quad (2)$$
$$x-1\geq 0\rightarrow x\geq 1 \quad (1)$$
$$(1)\cap(2)\rightarrow D_f=[1,82]$$

$x^2 - x - 2 > 0 \rightarrow (x+1)(x-2) > 0$ — ۳

x		-1		2	
	$+$		$-$		$+$

$\sqrt{x^2-1} \geq 0 \rightarrow x^2 - 1 \geq 0 \rightarrow x^2 \geq 1$

$(-\infty, -1) \cup (2, +\infty)$ ①

$x \geq 1$ or $x \leq -1$ ② $\xrightarrow{① \cap ②} \boxed{(-\infty, -1) \cup (2, +\infty)}$

— ۴

x		-2		b	
	$-$		$+$		$-$

$x = -2 \rightarrow -(-2)^2 + a(-2) + 3 = 0 \rightarrow 4 - 2a + 3 = 0$

$\rightarrow a = -\frac{1}{2} \rightarrow -x^2 + (-\frac{1}{2})x + 3 = 0 \xrightarrow{\times -1}$

$x^2 + \frac{1}{2}x - 3 = 0 \rightarrow 2x^2 + x - 6 = 0$ $\begin{cases} x = -2 \\ x = \frac{3}{2} \end{cases}$

$(2x+3)(x+2) = 0$

$a + b = -\frac{1}{2} + \frac{3}{2} = 1$

— ۵

$g(x) = \sqrt{f(x) - x} \geq 0 \rightarrow f(x) \geq x$

$f(x) \begin{cases} 3x - 2 & x \geq 1 \\ 2x + 3 & x < 1 \end{cases}$

x	1	2
y	1	4

x	1	0
y	5	3

$f(x) = \begin{cases} 3x - 2 \geq x \rightarrow x \geq 1 \\ 2x + 3 \geq x \rightarrow x \geq -3 \end{cases} \xrightarrow{\cap} D_g\ [-3, +\infty)$

— ۶

$f(a) = (a+1)(a+2) = 7a + 7$

$f(-2) = 3a + 2(-2) = 3a - 4$

$2f(a) = f(-2) + a \rightarrow 2(7a + 7) = 3a - 4 + a$

$14a + 14 = 4a - 4 \rightarrow 10a = -18 \rightarrow a = -1/8$

۷-

$$2+\sqrt{3} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} \to 2+\sqrt{3}$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3}) = \underbrace{\sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}}}_{m \text{ درنظر می‌گیریم}} + 2 + \overbrace{\sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}}}^{n} + 2$$

$$m^2 = 2-\cancel{\sqrt{3}} + 2+\cancel{\sqrt{3}} + 2\sqrt{\underbrace{(2-\sqrt{3})(2+\sqrt{3})}_{4-3=1}} \to 2+2+2 = 8 \to m=\sqrt{8}$$

$$n^2 = \frac{1}{2-\sqrt{3}} + \frac{1}{2+\sqrt{3}} + 2\left(\frac{1}{(\sqrt{2-\sqrt{3}})(\sqrt{2+\sqrt{3}})}\right) \to 2+\cancel{\sqrt{3}}+2-\cancel{\sqrt{3}}+2=8$$

$$n=\sqrt{8}$$

$$\sqrt{8}+\sqrt{8}+2+2 \to 2(2\sqrt{2})+4 = 4(\sqrt{2}+1)$$

۸-

$$x \to -x \to 2f(-x) - 3f(x) = 4(-x)^2 - (-x)$$

$$3\times \begin{cases} 2f(-x) - 3f(x) = 4x^2 + x \to 6f(-x) - 9f(x) = 12x^2 + 3x \\ 2f(x) - 3f(-x) = 4x^2 - x \to 4f(x) - 6f(-x) = 8x^2 - 2x \end{cases} \times 2$$

$$-5f(x) = 20x^2 + x \xrightarrow{\div 5}$$

$$f(x) = -4x^2 - \frac{1}{5}x$$

۹-

$$\text{if} \xrightarrow[x=0]{} 2f(0) = 3m-1$$

$$\text{if} \xrightarrow[x=-2]{} 6f(0) = 5m+15$$

$$9m-3 = 5m+15$$

$$4m = 18 \to m = \frac{9}{2} \to 4/5$$

$$\left.\begin{aligned} f(x) &= ax+b \\ f\left(\frac{1}{x}\right) &= \frac{a}{x}+b \end{aligned}\right\} f(x)+f\left(\frac{1}{x}\right) = ax+\frac{a}{x}+2b = 3x-12+\frac{3}{x}$$

$$a\left(x+\frac{1}{x}\right)+2b = 3\left(x+\frac{1}{x}\right)-12$$

$$a=3$$

$$2b=-12 \to b=-6 \longrightarrow f(x) = 3x-6 \to f(-1) = 3(-1)-6 = -9$$