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نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره ..... کلاس ....

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الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}$

$\frac{x-1}{x}-\frac{x}{x-1}\geq 0 \to \frac{x^2+1-2x-x^2}{x^2-x}=\frac{1-2x}{x(x-1)}\geq 0$

$x=\frac{1}{2}$, $x=0$, $x=1$

$$\begin{array}{c|c|c|c} & 0 & \frac{1}{2} & 1 \\ \hline + & - & + & - \end{array}$$

$D=(-\infty,0)\cup[\frac{1}{2},1)$

ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}$

$x+1\neq 0\to x\neq -1$

$x\neq 0$

$x-1\neq 0\to x\neq 1$

$x+2\neq 0\to x\neq -2$

$\frac{3}{x-1}+\frac{1}{x+2}\neq 0 \to \frac{3x+6+x-1}{x^2+x-2}\neq 0\to \frac{4x+5}{(x+2)(x-1)}$

$x\neq -\frac{5}{4}$, $x\neq -2$, $x\neq 1$

$D=\mathbb{R}-\{-2,-\frac{5}{4},-1,0,1\}$

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الف) $f(x)=\sqrt{((\frac{1}{3})^x-9)(32-4^x)}$

$((\frac{1}{3})^x-9)(32-4^x)\geq 0$

$(\frac{1}{3})^x-9=0\to (\frac{1}{3})^x=9\to (\frac{1}{3})^x=3^2\to 3^{-x}=3^2 \to x=-2$

$32-4^x=0\to 2^5=4^x\to 2^5=2^{2x}\to x=\frac{5}{2}$

$$\begin{array}{c|c|c} & -2 & \frac{5}{2} \\ \hline + & - & + \end{array}$$

$D=(-\infty,-2]\cup[\frac{5}{2},+\infty)$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3$

$\sqrt{y+1}=3-\sqrt[4]{x-1}$

$3-\sqrt[4]{x-1}\geq 0\to 3\geq \sqrt[4]{x-1}$

$\to 81\geq x-1\to x\leq 82$ ①

$\sqrt[4]{x-1}\xrightarrow{\text{شرط}} x-1\geq 0\to x\geq 1$ ②

$D_1\cap D_2=[1,82]$

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$f(x)=\dfrac{\log_4 (x^2-x-2)}{\sqrt{x^2-1}+1}$

$x^2-x-2>0\to (x-2)(x+1)>0\to$

$$\begin{array}{c|c|c} & -1 & 2 \\ \hline + & - & + \end{array}$$

$D_1=(-\infty,-1)\cup(2,+\infty)$

$\sqrt{x^2-1}+1\neq 0\to \sqrt{x^2-1}\neq -1$ هیچ گاه منفی نمی‌شود

$x^2-1\geq 0\to x^2\geq 1\to x\geq 1 \cup x\leq -1$

$D_2=[1,+\infty)\cup(-\infty,-1]$

$D_1\cap D_2=(-\infty,-1)\cup(2,+\infty)$

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$\sqrt{3+ax-x^2}\longrightarrow D=[-2,b]$ $\quad a+b=?$

$3+ax-x^2\geq 0$ $\quad x_2=b$, $-2=x_1$

$S=x_1+x_2=\frac{-b}{a}\to -2+b=\frac{-a}{-1}\to -2+b=a$ $\to \frac{3}{2}-2=a\to a=-\frac{1}{2}$

$P=x_1\times x_2=\frac{c}{a}\to -2\times b=\frac{3}{-1}\to 2b=3\to b=\frac{3}{2}$

$a+b=-\frac{1}{2}+\frac{3}{2}=1$

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$f(x)=\begin{cases}3x-2 & x\geq 1\\ 2x+3 & x<1\end{cases}$

① $x\geq 1\to 3x-2\geq x\to 2x\geq 2\to x\geq 1$

② $x<1\to 2x+3\geq x\to x\geq -3\to -3\leq x<1$

$g(x)=\sqrt{f(x)-x}\to f(x)\geq x$ $\quad D_{g(x)}=D_1\cup D_2=[-3,+\infty)$

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$$f(x)=\begin{cases}(a+1)(x+2) & x>1\\ 3a+2x & x\leq 1\end{cases}$$

$2f(5)=f(-2)+a \longrightarrow 2(7a+7)=3a-4+a \rightarrow 14a+14=3a-4+a$

$\rightarrow 10a=-18 \rightarrow \boxed{a=\frac{-18}{10}}$

$a=?$

$f(5)=(a+1)(\cancel{5+2})=7a+7$

$f(-2)=3a-4$

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$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2$

$f(2+\sqrt{3})=\sqrt{2+\sqrt{3}}+\cancel{\frac{1}{\sqrt{2+\sqrt{3}}}}+2=\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}+2$

$f(2-\sqrt{3})+f(2+\sqrt{3})=?$

$f(2-\sqrt{3})=\sqrt{2-\sqrt{3}}+\cancel{\frac{1}{\sqrt{2-\sqrt{3}}}}+2=\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}+2$

$\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}+2+\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}+2=2(\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}})+4=\boxed{2\sqrt{6}+4}$

$(\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}})^2=2+\sqrt{3}+2-\sqrt{3}+2\sqrt{4-3}=6 \Rightarrow \sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}=\sqrt{6}$

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$2f(x)-3f(-x)=4x^2-x \qquad f(x)=?$

$x\rightarrow \quad 2f(x)-3f(-x)=4x^2-x \xrightarrow{\times 2}$

$-x\rightarrow \quad 2f(-x)-3f(x)=4x^2+x \xrightarrow{\times 3}$

$\begin{cases}4f(x)-6f(-x)=8x^2-2x\\ 6f(-x)-9f(x)=12x^2+3x\end{cases}$

$-5f(x)=20x^2+x \longrightarrow f(x)=\frac{-20x^2-x}{5}=\boxed{-4x^2-\frac{1}{5}x}$

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$(x+2)f(x)-3xf(x+2)=4x^2-mx+3m-1 \qquad f(0)=?$

$x=0\rightarrow 2f(0)=3m-1 \xrightarrow{\times 3} 6f(0)=9m-3$

$x=-2\rightarrow 6f(0)=16+2m+3m-1=15+5m \longrightarrow 6f(0)=15+5m$

$\rightarrow 9m-3=15+5m \rightarrow 4m=18 \rightarrow m=\frac{18}{4}=\frac{9}{2}$

$m=\frac{9}{2} \rightarrow 2f(0)=3(\frac{9}{2})-1=\frac{27}{2}-1=\frac{25}{2} \Rightarrow f(0)=\frac{\frac{25}{2}}{2}=\boxed{\frac{25}{4}}$

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روش ۱

$f(x)+f(\frac{1}{x})=\frac{3x^2-12x+3}{x} \qquad f(-1)=?$

$x=-1\rightarrow f(-1)+f(-1)=\frac{3+12+3}{-1} \rightarrow 2f(-1)=-18 \Rightarrow \boxed{f(-1)=-9}$

روش ۲

$ax+b+a(\frac{1}{x})+b=\frac{3x^2-12x+3}{x} \rightarrow ax+\frac{a}{x}+2b=3x-12+\frac{3}{x} \Rightarrow a=3,\ b=-6$

$f(x)=ax+b \rightarrow f(x)=3x-6 \longrightarrow f(-1)=3\times -1-6=\boxed{-9}$

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