نام و نام خانوادگی : بهار جواهری .... پاسخنامه تشریحی تکلیف شماره .... کلاس ....

1

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} = \frac{(x-1)^2 - x^2}{x(x-1)} = \frac{\cancel{x^2} - 2x + 1 - \cancel{x^2}}{x(x-1)} = \frac{-2x+1}{x(x-1)} \geq 0$ ، $x \to \frac{1}{2}$ ، $x \to 0,\ 1$

$0 \quad \frac{1}{2} \quad 1$: $+ \quad | \quad - \quad \phi \quad + \quad | \quad -$ $\rightsquigarrow (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $\dfrac{\frac{1}{x+1} - \frac{2}{x}}{\frac{3}{x-1} + \frac{1}{x+2}} \neq 0$ ... $\frac{3}{x-1} + \frac{1}{x+2} \neq 0 \to \frac{3x+6+x-1}{(x-1)(x-2)} \neq 0 \to \frac{4x+5}{(x-1)(x-2)} \neq 0$ ، $x \neq \frac{-5}{4}$

2

الف) $\sqrt{\left((\frac{1}{3})^x - 9\right)\left(32 - 4^x\right)} \geq 0$

$(\frac{1}{3})^x = 9 \to x = -2$

$32 = 4^x \to 2^{2x} = 2^5 \to 2x = 5 \to x = 2.5$

$-2 \quad 2.5$: $+ \quad \phi \quad - \quad \phi \quad +$ $\to (-\infty, -2] \cup [2.5, +\infty)$

ب) $\sqrt[4]{x-1} + \sqrt{y+1} = 3$ ، $x - 1 \geq 0 \to x \geq 1$ ، $\sqrt{y+1} = 3 - \sqrt[4]{x-1} \to 3 - \sqrt[4]{x-1} \geq 0 \to \sqrt[4]{x-1} \leq 3 \to x - 1 \leq 81 \to x \leq 82$

$y+1 \in [0, 9] \to y \in [-1, 8]$

$x \to [1, 82]$

3

$\dfrac{\log_4 (x^2 - x - 2)}{\sqrt{x^2-1} + 1}$

1) $x^2 - x - 2 > 0 \to (x-2)(x+1) > 0$ ، $-1 \quad 2$: $+ \quad \phi \quad - \quad \phi \quad +$ $\to (-\infty, -1] \cup [2, +\infty)$

2) $x^2 - 1 \geq 0 \to$ $-1 \quad +1$: $+ \quad \phi \quad - \quad \phi \quad +$ $\to (-\infty, -1] \cup [1, +\infty)$

3) $\sqrt{x^2-1} + 1 \neq 0 \to \sqrt{x^2-1} \neq -1 \to$ همواره برقرار است

$1 \cap 2 \cap 3 \to (-\infty, -1] \cup [2, +\infty)$

4

$\sqrt{3 + ax - x^2} \geq 0 \to -x^2 + ax + 3 \geq 0 \to x^2 - ax - 3 \leq 0 \to x = \frac{a \pm \sqrt{a^2+12}}{2}$

$D = \left[\frac{a - \sqrt{a^2+12}}{2}, \frac{a + \sqrt{a^2+12}}{2}\right]$ ، $\frac{a - \sqrt{a^2+12}}{2} = -2 \to a - \sqrt{a^2+12} = -4$

$\frac{-\frac{1}{2} + \sqrt{\frac{1}{4} + 12}}{2} = b = \frac{a + \sqrt{a^2+12}}{2} \leftarrow a = -\frac{1}{2} \leftarrow (a+4)^2 = a^2 + 12 \leftarrow a \geq -4 \leftarrow a + 4 = \sqrt{a^2+12} \geq 0$ ، $\frac{3}{2}$

$\to a + b = -\frac{1}{2} + \frac{3}{2} = 1$

5

$f(x) \begin{cases} 3x - 2 & x \geq 1 \\ 2x + 3 & x < 1 \end{cases}$ $\quad g(x) = \sqrt{f(x) - x}$

$f(1) = 1 \to g(x) = 0$ $\quad f(-3) = 0 \to g(x) = \sqrt{3}$

$f(0) = 3 \to g(x) = \sqrt{3}$ $\quad f(-4) = -5 \to g(x) = \sqrt{-1}$ ✗

$f(-1) = 1 \to g(x) = \sqrt{2}$

$\to D_g = [-3, +\infty)$

$\begin{cases}(a+1)(x+2) & x>1\\ 3a+2x & x\leq 1\end{cases}$ $\quad 2f(5)=f(-2)+a \quad a=?$ $f(5)=(a+1)(7) \to 7a+7 \to 2f(5)=14a+14$ $f(-2)=3a-4 \to 14a+14=3a-4+a \to a=-\frac{18}{10}$	۶
$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2 \quad f(2-\sqrt{3})+f(2+\sqrt{3})=A$	۷
$x=x \begin{cases}2f(x)-3f(-x)=4x^2-x\\ 2f(-x)-3f(x)=4x^2+x\end{cases}$ $x=-x$ $\to \begin{cases}4f(x)-\cancel{6f(-x)}=8x^2-2x\\ \cancel{6f(-x)}-9f(x)=12x^2+3x\end{cases} \to$ $-5f(x)=20x^2+x \to 5f(x)=-20x^2-x \to f(x)=-4x^2-\frac{x}{5}$	۸
$(x+2)f(x)-3xf(x+2)=4x^2-mx+3m-1 \quad f(0)=?$ $x=0 \to (0+2)f(0)-3\times 0\times f(2)=4(0)-m(0)+3m+1 \to f(0)=\frac{3m-1}{2}$ $x=-2 \to (0)f(-2)-\underbrace{3(-2)f(0)}_{6}=16+2m+3m-1 \to f(0)=\frac{15+5m}{6}$ $\frac{15+5m}{3\cancel{6}}=\frac{3m-1}{\cancel{2}} \to m=\frac{18}{4}=\frac{9}{2} \to f(0)=\frac{25}{4}$ $9m-3=15+5m$ $4m=18$	۹
$f(x)+f(\frac{1}{x})=\frac{3x^2-11x+3}{x} \quad f(-1)$ $f(x)=ax+b \to f(\frac{1}{x})=\frac{a}{x}+b \quad \frac{a}{x}+b+ax+b=\frac{a+ax^2+2bx}{x}$ $\to \frac{ax^2+2bx+a}{\cancel{x}}=\frac{3x^2-11x+3}{\cancel{x}} \to \begin{matrix}a=3\\ b=-6\end{matrix} \to f(-1)=-a+b=-3-6=-9$	۱۰