نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

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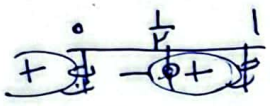
الف) $f(x)=\sqrt{\frac{x-1}{x}}-\frac{x}{x-1}=\frac{(x-1)^2-x^2}{x(x-1)}=\frac{-2x+1}{x(x-1)}\geq 0$



$\rightarrow (-\infty,0)\cup[\frac{1}{2},1)$

ب) $\frac{\frac{x}{x+1}-\frac{1}{3}}{\frac{1}{x-1}+\frac{1}{x+2}}\neq 0 \quad \frac{1}{x-1}+\frac{1}{x+2}\neq 0 \rightarrow \frac{3x+1+x-1}{(x-1)(x+2)}\neq 0 \rightarrow \frac{4x}{(x-1)(x+2)}\neq 0 \rightarrow x\neq 0$

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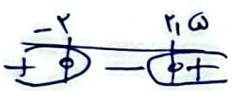
الف) $\sqrt{\left(\left(\frac{1}{3}\right)^x-9\right)(3^x-3^2)}\geq 0$

$\left(\frac{1}{3}\right)^x=9=3^{-2} \rightarrow x=-2$

$3^x=3^2 \rightarrow x=2$



$\rightarrow (-\infty,-2]\cup[2,+\infty)$

ب) $\sqrt[3]{x-1}+\sqrt{x+1}=3$

$\sqrt{x+1}=3-\sqrt[3]{x-1} \rightarrow 3-\sqrt[3]{x-1}>0 \rightarrow \sqrt[3]{x-1}<3 \rightarrow x-1<27 \rightarrow x<28$

$x+1\in[0,29] \rightarrow x\in[-1,28]$

$x\rightarrow[-1,28]$

$1\leq x<28$

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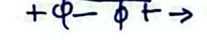
$\log_{\frac{1}{3}}^{x^2-x-2}$ / $\sqrt{x^2-1}+1$

1) $x^2-x-2>0 \rightarrow (x-2)(x+1)>0$  $\rightarrow (-\infty,-1]\cup[2,+\infty)$

2) $x^2-1\geq 0$ $\rightarrow (-\infty,-1]\cup[1,+\infty)$

3) $\sqrt{x^2-1}+1\neq 0 \rightarrow \sqrt{x^2-1}\neq -1 \rightarrow$ همواره برقرار است

$1\wedge 2\wedge 3 \rightarrow (-\infty,-1]\cup[2,+\infty)$

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$\sqrt{3+ax-x^2}\geq 0 \rightarrow -x^2+ax+3\geq 0 \rightarrow x^2-ax-3\leq 0 \rightarrow x=\frac{a\pm\sqrt{a^2+12}}{2}$

$D=\left[\frac{a-\sqrt{a^2+12}}{2},\frac{a+\sqrt{a^2+12}}{2}\right]$

$\frac{a-\sqrt{a^2+12}}{2}=-2 \rightarrow a-\sqrt{a^2+12}=-4 \rightarrow a+4=\sqrt{a^2+12}$

$\rightarrow a\geq -4 \leftarrow a+4\geq 0$

$a+4=\sqrt{a^2+12} \rightarrow (a+4)^2=a^2+12 \rightarrow a=-\frac{1}{2}$

$b=\frac{a+\sqrt{a^2+12}}{2}=\frac{-\frac{1}{2}+\sqrt{\frac{1}{4}+12}}{2}=\frac{3}{4}$

$\rightarrow a+b=-\frac{1}{2}+\frac{3}{4}=\frac{1}{4}$

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$f(x)\begin{cases}x^2-2 & x\leq 1\\ x+3 & x>1\end{cases}$ $\quad g(x)=\sqrt{f(x)-x}$

$f(1)=1 \rightarrow g(1)=0$

$f(0)=3 \rightarrow g(0)=\sqrt{3}$

$f(-1)=1 \rightarrow g(-1)=\sqrt{3}$

$f(-3)=6 \rightarrow g(-3)=\sqrt{9}$

$f(-2)=2 \rightarrow g(-2)=\sqrt{4}$

$\rightarrow D_g=[-3,+\infty)$

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$f(x)=\begin{cases}(a+1)(x+2) & x>1\\ 3a+2x & x\le 1\end{cases}$ $\quad 2f(5)=f(-2)+a \quad a=?$

$f(5)=(a+1)(7) \to 7a+7 \to 2f(5)=14a+14$

$f(-2)=3a-4 \to 14a+14=3a-4+a \to a=-\frac{18}{10}$

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$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2 \quad f(2-\sqrt{3})+f(2+\sqrt{3})=A$

$(2-\sqrt{3})(2+\sqrt{3})=1$ عکس یکدیگرند

جواب نهایی $=2\sqrt{6}+4$

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$x\to x: \; 2f(x)-3f(-x)=4x^2-x$

$x\to -x: \; 2f(-x)-3f(x)=4x^2+x$

$\to \begin{cases}4f(x)-6f(-x)=8x^2-2x\\ 6f(-x)-9f(x)=12x^2+3x\end{cases}$

$-5f(x)=20x^2+x \to 5f(x)=-20x^2-x \to f(x)=-4x^2-\frac{x}{5}$

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$(x+2)f(x)-3xf(x+2)=4x^2-mx+3m-1 \quad f(0)=?$

$x=0 \to (0+2)f(0)-3\times 0\times f(2)=4(0)-m(0)+3m+1 \to f(0)=\frac{3m-1}{2}$

$x=-2 \to (0)f(-2)-3(-2)f(0)=16+2m+3m-1 \to f(0)=\frac{15+5m}{6}$

$\frac{15+5m}{6}=\frac{3m-1}{2} \to m=\frac{18}{4}=\frac{9}{2} \to f(0)=\frac{25}{4}$

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$f(x)+f(\frac{1}{x})=\frac{3x^2-12x+3}{x} \quad f(-1)$

$f(x)=ax+b \to f(\frac{1}{x})=\frac{a}{x}+b \qquad \frac{a}{x}+b+ax+b=\frac{a+ax^2+2bx}{x}$

$\to \frac{ax^2+2bx+a}{x}=\frac{3x^2-12x+3}{x} \to a=3,\; b=-6 \to f(-1)=-a+b=-3-6=-9$

$x+1 \neq 0$, $x \neq 0$, $x-1 \neq 0$, $x+2 \neq 0$, $\frac{3}{x-1} + \frac{1}{x+2} \neq 0$ (ب 1

$D_f = \mathbb{R} - \left\{ \pm 1, 0, -2, -\frac{5}{2} \right\}$