یازدهم دختر B — شماره تکلیف = ۴ — ستوده مرادی

1) $f(x) = \sqrt{\dfrac{x-1}{x} - \dfrac{x}{x-1}}$ $\quad D_{f_1} = (-\infty, 0) \cup [\tfrac{1}{2}, 1)$

$\dfrac{x-1}{x} - \dfrac{x}{x-1} \geq 0$

$0 \quad \tfrac{1}{2} \quad 1$

$+ \ \text{تن} \ - \ 0 \ + \ \text{تن} \ -$

$\dfrac{(x-1)^2 - x^2}{x^2-x} = \dfrac{x^2+1-2x-x^2}{x^2-x} \rightarrow \dfrac{-2x+1}{x(x-1)} \geq 0$

2) $f(x) = \dfrac{\dfrac{1}{x+1} - \dfrac{2}{x}}{\dfrac{3}{x-1} + \dfrac{1}{x+2}}$ $\rightarrow x \neq -1, \ x \neq 0$

$\dfrac{3}{x-1} + \dfrac{1}{x+2} \neq 0$

$\dfrac{3(x+2) + x - 1}{(x-1)(x+2)} \rightarrow \dfrac{4x+5}{(x-1)(x+2)}$

$D_{f_2} = \mathbb{R} - \{-2, -\tfrac{5}{4}, -1, 0, 1\}$

1

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1) $\sqrt{\left((\tfrac{1}{3})^x - 9\right)\left(32 - 4^x\right)}$

$(\tfrac{1}{3})^x - 9 = 0 \longrightarrow (\tfrac{1}{3})^x = 9 \rightarrow x = -2$

$32 - 4^x = 0 \rightarrow 2^{2x} = 2^5 \rightarrow x = \tfrac{5}{2}$

$-2 \quad \tfrac{5}{2}$

$+ \ 0 \ - \ 0 \ +$

$D_{f_1} = (-\infty, -2] \cup [\tfrac{5}{2}, +\infty)$

2) $\sqrt[4]{x-1} + \sqrt{y+1} = 3$

$\sqrt{y+1} = 3 - \sqrt[4]{x-1} \Rightarrow 3 - \sqrt[4]{x-1} \geq 0$

$\sqrt[4]{x-1} \leq 3 \xrightarrow{\text{توان}} x - 1 \leq 81$

$x - 1 \geq 0 \rightarrow x \geq 1 \qquad x \leq 82$

$D_{f_2} = [1, 82]$

2

$f(x) = \dfrac{\log_4^{(x^2-x-2)}}{\sqrt{x^2-1} + 1}$ $\quad x^2 - x - 2 > 0 \rightsquigarrow (x-2)(x+1) > 0$

$x = 2 \quad x = -1$

$-1 \quad 2$

$+ \ 0 \ - \ 0 \ +$

$-1 \quad 1 \quad 2$

$D_f = (-\infty, -1) \cup (2, +\infty)$

$x^2 - 1 \geq 0$

$x^2 \geq 1 \rightarrow x \geq 1$

$x \leq -1$

3

$f(x) = \sqrt{3 + ax - x^2}$ $\quad -x^2 + ax + 3 \geq 0$

$-2 \quad b$

$+ \ 0 \ - \ 0 \ +$

$D_f = [-2, b]$ $\quad x^2 - ax - 3 \leq 0$

$(x+2)(x-b) \longrightarrow (x+2)(x - \tfrac{3}{2}) \Rightarrow x^2 + \tfrac{1}{2}x - 3 \leq 0$

$a + b = ?$ $\quad -2 \times b = -3 \qquad a = -\tfrac{1}{2}$

$b = -\tfrac{3}{-2} = \tfrac{3}{2}$ $\qquad a + b = \tfrac{3}{2} - \tfrac{1}{2} = \tfrac{2}{2} = \boxed{1}$

4

$f(x) = \begin{cases} 3x - 2, & x \geq 1 \\ 2x + 3, & x < 1 \end{cases}$ $\quad \rightarrow 3x - 2 \geq x$

$2x \geq 2$

$x \geq 1$

برای اعداد بزرگتر از ۱ همواره برقرار

ولی برای اعداد کوچکتر از ۱

از ۳- به بعد برقرار است

$g(x) = \sqrt{f(x) - x}$ $\quad 2x + 3 \geq x$

$x + 3 \geq 0$

$x \geq -3$

$D_g = [-3, +\infty)$

5

$D_g = ?$ $\qquad f(x) - x \geq 0 \rightarrow f(x) \geq x$

$$f(x) = \begin{cases} (a+1)(x-2) & , x>1 \\ 3a+2x & , x\leq 1 \end{cases}$$

$2f(5) = f(-2)+a \longrightarrow 2(a+1)(5+2) = 3a+2(-2)+a$

$a = ?$

$2(5a+2a+5+2) = 4a-4$

$14a+14 = 4a-4$

$10a = -18$

$a = -18/10 = \boxed{-9/5}$

6

$f(x) = \sqrt{x} + \dfrac{1}{\sqrt{x}} + 2$

$f(\underbrace{\overbrace{2-\sqrt{3}}^{x_1}}) + f(\underbrace{\overbrace{2+\sqrt{3}}^{x_2}}) = ?$

معکوس هم هستند چون حاصل ضرب ۱ می‌شود

$\sqrt{x_1} + \overset{\sqrt{x_2}}{\dfrac{1}{\sqrt{x_1}}} + 2 + \sqrt{x_2} + \overset{\sqrt{x_1}}{\dfrac{1}{\sqrt{x_2}}} + 2 = 4 + 2\sqrt{x_1} + 2\sqrt{x_2}$ 7

$\longrightarrow \boxed{4 + 2\sqrt{2-\sqrt{3}} + 2\sqrt{2+\sqrt{3}}}$

$2f(x) - 3f(-x) = 4x^2 - x$

$f(x) = ?$

$(2f(x) - 3f(-x) = 4x^2 - x) \times 2$

$(2f(-x) - 3f(x) = 4x^2 + x) \times 3$

$4f(x) - 6f(-x) = 8x^2 - 2x$

$6f(-x) - 9f(x) = 12x^2 + 3x$ 8

$-5f(x) = 20x^2 + x$

$f(x) = \dfrac{20x^2 + x}{-5} \Rightarrow \boxed{-4x^2 - \dfrac{x}{5}}$

$(x+2)f(x) - 3x f(x+2) = 4x^2 - mx + 3m - 1$

$f(0) = ?$

$x = 0 \rightarrow 2f(0) = 3m-1$

$f(0) = \dfrac{3m-1}{2}$

$\dfrac{3m-1}{2} = \dfrac{5m+15}{6}$

$18m - 6 = 10m + 30$ 9

$8m = 36 \longrightarrow m = 36/8 = 9/2$

$x = -2 \Rightarrow 6f(0) = 16 + 2m + 3m - 1$

$6f(0) = 15 + 5m \longrightarrow f(0) = \dfrac{5m+15}{6}$

$\dfrac{3 \times 9/2 - 1}{2} = \dfrac{25/2}{2} \Rightarrow f(0) = \boxed{\dfrac{25}{4}}$

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f تابع خطی $\rightarrow ax+b$

$f(x) + f\left(\dfrac{1}{x}\right) = \dfrac{3x^2 - 12x + 3}{x} \longrightarrow x = -1 \Rightarrow 2f(-1) = \dfrac{3+12+3}{-1}$ 10

$f(-1) = ?$

$2f(-1) = -18$

$f(-1) = \boxed{-9}$