بازدهم دفتر B — شماره کلاس = ۴ — ستوده مرادی — ۱۹٫۵

1) $f_{(x)} = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$ $D_{f_1} = (-\infty, 0) \cup [\frac{1}{2}, 1)$

$\frac{x-1}{x} - \frac{x}{x-1} \geq 0$

$\frac{(x-1)^2 - x^2}{x^2 - x} = \frac{x^2+1-2x-x^2}{x^2-x} \rightarrow \frac{-2x+1}{x(x-1)} \geq 0$

2) $f_{(x)} = \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{3}{x-1} + \frac{1}{x+2}}$ $\rightarrow x \neq -1,\ x \neq 0$

$\frac{3}{x-1} + \frac{1}{x+2} \neq 0$

$\frac{3(x+2) + x-1}{(x-1)(x+2)} \rightarrow \frac{4x+5}{(x-1)(x+2)}$

$D_{f_2} = \mathbb{R} - \{-2, -\frac{5}{4}, -1, 0, 1\}$

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1) $\sqrt{((\frac{1}{3})^x - 9)(32 - 4^x)}$

$(\frac{1}{3})^x - 9 = 0 \rightarrow (\frac{1}{3})^x = 9 \rightarrow x = -2$

$32 - 4^x = 0 \rightarrow 2^{2x} = 2^5 \rightarrow x = \frac{5}{2}$

$D_{f_1} = (-\infty, -2] \cup [\frac{5}{2}, +\infty)$

2) $\sqrt[4]{x-1} + \sqrt{y+1} = 3$

$\sqrt{y+1} = 3 - \sqrt[4]{x-1} \Rightarrow 3 - \sqrt[4]{x-1} \geq 0$

$\sqrt[4]{x-1} \leq 3 \xrightarrow{\text{توان}} x-1 \leq 81$

$x-1 \geq 0 \rightarrow x \geq 1$ $\quad x \leq 82$

$D_{f_2} = [1, 82]$

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$f_{(x)} = \frac{\log_4^{(x^2-x-2)}}{\sqrt{x^2-1} + 1}$

$x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0$ $\quad x = 2,\ x = -1$

$x^2 - 1 \geq 0$

$x^2 \geq 1 \rightarrow x \geq 1,\ x \leq -1$

$D_f = (-\infty, -1) \cup (2, +\infty)$

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$f_{(x)} = \sqrt{4 + ax - x^2}$

$D_f = [-2, b]$

$a + b = ?$

$-x^2 + ax + 4 \geq 0$

$x^2 - ax - 4 \leq 0$

$(x+2)(x-b) \rightarrow (x+2)(x-\frac{3}{2}) \Rightarrow x^2 + \frac{1}{2} - 3 \leq 0$

$-2 \times b = -4$

$b = -\frac{3}{-2} = \frac{3}{2}$

$a = -\frac{1}{2}$

$a + b = \frac{3}{2} - \frac{1}{2} = \frac{2}{2} = 1$

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$f_{(x)} = \begin{cases} 3x-2, & x \geq 1 \\ 2x+3, & x < 1 \end{cases}$

$\rightarrow 3x - 2 \geq x$

$2x \geq 2$

$x \geq 1$

$g_{(x)} = \sqrt{f_{(x)} - x}$

$2x + 3 \geq x$

$x + 3 \geq 0$

$x \geq -3$

برای اعداد بزرگتر از ۱ همواره برقرار

ولی برای اعداد کوچکتر از ۱

از ۳- به بعد برقرار است

$D_g = [-3, +\infty)$

$D_g = ?$ $\quad f_{(x)} - x \geq 0 \rightarrow f_{(x)} \geq x$

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$$f(x)=\begin{cases}(a+1)(x-2) & x>1\\ 2a+2x & x\le 1\end{cases}$$

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$2f(3)=f(-2)+a \rightarrow 2(a+1)(3+2)=2^2a+2(-2)+a$

$a=?$

$2(5a+2a+3+2)=4a-4$

$14a+14=4a-4$

$10a=-18$

$a=-18/10=\boxed{-9/5}$

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$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2$

$f(\underbrace{x_1}_{2-\sqrt{2}})+f(\underbrace{x_2}_{2+\sqrt{2}})=?$

$\sqrt{x_1}+\frac{1}{\sqrt{x_1}}+2+\sqrt{x_2}+\frac{1}{\sqrt{x_2}}+2=4+2\sqrt{x_1}+2\sqrt{x_2}$

$\rightarrow \boxed{4+2\sqrt{2-\sqrt{2}}+2\sqrt{2+\sqrt{2}}}=4+2\sqrt{4}$

$2\left(\sqrt{2-\sqrt{2}}+\sqrt{2+\sqrt{2}}\right)$

A , $A^2=4$

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$2f(x)-3f(-x)=4x^2-x$

$f(x)=?$

$(2f(x)-3f(-x)=4x^2-x)\times 2$

$(2f(-x)-3f(x)=4x^2+x)\times 3$

$4f(x)-6f(-x)=8x^2-2x$

$6f(-x)-9f(x)=12x^2+3x$

$-5f(x)=20x^2+x$

$f(x)=\frac{20x^2+x}{-5}\Rightarrow \boxed{-4x^2-\frac{x}{5}}$

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$(x+2)f(x-1)-2xf(x+2)=4x^2-mx+3m-1$

$f(0)=?$

$x=0 \rightarrow 2f(0)=3m-1$

$f(0)=\frac{3m-1}{2}$

$x=-2 \Rightarrow -4f(0)=16+2m+3m-1$

$-4f(0)=15+5m \rightarrow f(0)=\frac{5m+15}{-4}$

$\frac{3m-1}{2}=\frac{5m+15}{-4}$

$15m-4=10m+30$

$5m=34 \rightarrow m=34/5=9/2$

$\frac{3\times 9/2-1}{2}=\frac{25}{2\times 2}\Rightarrow f(0)=\frac{25}{4}$

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f خطی $\rightarrow ax+b$

$f(x)+f\left(\frac{1}{x}\right)=\frac{3x^2-12x+3}{x}$ $\rightarrow x=-1 \Rightarrow 2f(-1)=\frac{3+12+3}{-1}$

$f(-1)=?$

$2f(-1)=-18$

$f(-1)=\boxed{-9}$