نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره ۲ ..... کلاس ....

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الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}} \Rightarrow \frac{x-1}{x}-\frac{x}{x-1}\geq 0 \Rightarrow \frac{x^2+1-2x-x^2}{x^2-x}\geq 0 \Rightarrow \frac{1-2x}{x^2-x}\geq 0 \Rightarrow x\leq \frac{1}{2}$

$\to x(x-1)=0$, $x\neq 1,0$

$$\begin{array}{ccccccc} & 0 & & \frac{1}{2} & & 1 & \\ + & & - & & + & & - \end{array} \Rightarrow D_f=(-\infty,0)\cup[\frac{1}{2},1)$$

ب) $f(x)=\dfrac{\frac{x-2x+2}{x^2+x}}{\frac{3x+4+x-1}{x^2+x-2}} \Rightarrow \frac{-x+2}{x(x+1)}$ $x\neq 0$, $x\neq -1$

$D_f=\mathbb{R}-\{-2,-1,-\frac{5}{4},0,1\}$

$\to \frac{4x+5}{(x-1)(x+2)}$ $4x=-5 \Rightarrow x=-\frac{5}{4}$

$x=+1$ $x=-2$

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الف) $f(x)=\sqrt{((\frac{1}{3})^x-9)(32-4^x)} \Rightarrow (\frac{1}{3})^x-9\geq 0 \Rightarrow (\frac{1}{3})^x\geq 3^2 \Rightarrow 3^{-x}\geq 3^2 \Rightarrow x\leq -2$

$32-4^x\geq 0 \Rightarrow 2^5\geq 2^{2x} \Rightarrow 5\geq 2x \Rightarrow x\leq \frac{5}{2}$

$$\begin{array}{ccccc} & -2 & & \frac{5}{2} & \\ + & & - & & + \end{array} \quad D_f=(-\infty,-2]\cup(\frac{5}{2},+\infty)$$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3 \Rightarrow \sqrt{y+1}=(3-\sqrt[4]{x-1})^2 \Rightarrow y+1=9-\sqrt{x-1} \Rightarrow \sqrt{x-1}\geq 0$, $x\geq 1$

$\sqrt{y+1}\geq 0 \Rightarrow$ میدانیم $\sqrt{y+1}=3-\sqrt[4]{x-1} \Rightarrow 3-\sqrt[4]{x-1}\geq 0$ $x\leq 82$

$D_f=[-1,82]$

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$f(x)=\frac{\log_2(x^2-x-2)}{\sqrt{x^2-1}+1}$ $>0 \Rightarrow x^2-x-2 \Rightarrow (x+1)(x-2)>0$

$x=-1$ $x=2$

$$\begin{array}{ccccc} & -1 & & 2 & \\ + & & - & & + \end{array}$$

$(-\infty,-1)\cup(2,+\infty)$ I

$\to \sqrt{x^2-1}\geq 0 \Rightarrow x^2\geq 1 \Rightarrow x\geq 1$ و $x\leq -1$

$(-\infty,-1]\cup[1,+\infty)$ II

$I\cap II \Rightarrow D_f=(-\infty,-1)\cup(2,+\infty)$

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$\sqrt{3+ax-x^2} \Rightarrow x=-2 \Rightarrow \sqrt{3-2a-4} \Rightarrow \sqrt{-1-2a}=0 \Rightarrow \boxed{a=-\frac{1}{2}}$

$D_f=[-2,b]$ $\xrightarrow{a=-\frac{1}{2}}$ $\sqrt{-x^2-\frac{x}{2}+3} \Rightarrow \Delta\geq 0 \Rightarrow \frac{1}{4}+\frac{48}{4}=\frac{49}{4}$

$x_1=-2 \Rightarrow$ داریم

$x_2=\boxed{+\frac{3}{2}=b}$ پس

$a+b=-\frac{1}{2}+\frac{3}{2}=\frac{2}{2}=1$

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$f(x)=\begin{cases}3x-2 & x\geq 1\\ 2x+3 & x<1\end{cases}$

① $\sqrt{3x-2-x} \Rightarrow \sqrt{2x-2} \Rightarrow x\geq 1$

② $\sqrt{2x+3-x} \Rightarrow \sqrt{x+3}\geq 0 \Rightarrow x\geq -3$

$g(x)=\sqrt{f(x)-x} \Rightarrow f(x)\geq x$

$D_f=[-3,+\infty)$

$f(x)=\begin{cases}(x+1)(x+2) & x>1\\ 3a+2x & x\leq 1\end{cases}$ $\quad 2f(5)=f(-2)+a$ $\Rightarrow -4+4a=14a+14 \Rightarrow -11=11a \Rightarrow a=-1$	۶
$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2 \Rightarrow \frac{x+1+2\sqrt{x}}{\sqrt{x}}$ $\quad f(2-\sqrt{3})+f(2+\sqrt{3})=?$ $\Rightarrow 2-\sqrt{3}+\frac{1}{2-\sqrt{3}}\cdot\frac{2+\sqrt{3}}{2+\sqrt{3}}+2+2+\sqrt{3}+\frac{1}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}}+2 \Rightarrow 2+\sqrt{3}+\frac{2-\sqrt{3}}{4-3}+2+$ $2-\sqrt{3}+\frac{2+\sqrt{3}}{4-3}+2 \Rightarrow 8+4=12$	۷
$2f(x)-3f(-x)=4x^2-x \xRightarrow{x=-x} 2f(-x)-3f(x)=4x^2+x \Rightarrow f(x)-f(-x)=8x^2$ $f(-x)=-8x^2-f(x) \Rightarrow 2f(x)-3(-8x^2-f(x))=4x^2-x$ $\Rightarrow 5f(x)+24x^2=4x^2-x \Rightarrow 5f(x)=4x^2-24x^2-x \Rightarrow -20x^2-x$ $5f(x)=-20x^2-x \Rightarrow f(x)=-4x^2-\frac{x}{5}$	۸
$(x+2)f(x)-4xf(x+2)=4x^2-mx+3m-1$ $x=0 \Rightarrow 2f(0)-f(2)\times 0=3m-1$ $x=-2 \Rightarrow 0(f(-2))+6f(0)=16+5m-1$ $-6f(0)=15-5m$ $6f(0)=3m-1$ $\frac{4}{4}f(0)=\frac{14+2m}{4}$ $\Rightarrow f(0)=4+\frac{m}{2} \Rightarrow f(0)=\frac{14}{4}+\frac{9}{4}=\boxed{\frac{25}{4}}$ $m \Rightarrow \frac{15+5m}{6}=\frac{3m-1}{2} \Rightarrow 9m-3=15+5m$ $m=\frac{9}{2}$	۹
$f(x)+f(\frac{1}{x})=\frac{3x^2-12x+3}{x}$ $f(x)=ax+b$ $f(\frac{1}{x})=\frac{a}{x}+bx+ax^2+bx=\frac{3x^2-12x+3}{x} \Rightarrow ax^2+2bx+a=3x^2-12x+3$ $\Rightarrow 2bx=-12x \Rightarrow b=-6$ $\quad f(x)=ax+b \Rightarrow f(x)=3a-6$ $\quad \boxed{f(-1)=-9}$ $a=3$	۱۰