(تکلیف شماره) (فاطمه شکرانی) (یازدهم دفتر B)

۱

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$

$\frac{x-1}{x} - \frac{x}{x-1} \geq 0$

$\frac{x^2+1-2x-x^2}{x^2-x} = \frac{1-2x}{x(x-1)} \geq 0$

0		$\frac{1}{2}$		1	
$+$		$-$	$+$		$-$

$D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \dfrac{\frac{1}{x+1} - \frac{2}{x}}{\frac{3}{x-1} + \frac{1}{x+2}}$ $\quad -1,\ 0,\ 1,\ -2$

$x \neq 0, -1, 1, -2$

$\frac{3}{x-1} + \frac{1}{x+2} \neq 0 \quad \frac{3x+6+x-1}{(x-1)(x+2)} = \frac{4x+5}{(x-1)(x+2)} \to x \neq -\frac{5}{4}$

$D_f = R - \{-2, -\frac{5}{4}, -1, 0, 1\}$

۲

الف) $f(x) = \sqrt{((\frac{1}{3})^x - 9)(32 - 4^x)} \to 2^5 = 32 = 2^{2x} \quad x = \frac{5}{2}$

$((\frac{1}{3})^x - 9)(32 - 4^x) \geq 0$

$3^{-x} - 9 = 0 \quad 3^{-x} = 3^2 \quad x = -2$

-2		2.5	
$+$	$-$		$+$

$D_f = R - (-2, 2.5)$

ب) $\sqrt[4]{x-1} + \sqrt{y+1} = 3 \quad x-1 \geq 0 \quad x \geq 1$

$\sqrt{y+1} = 3 - \sqrt[4]{x-1} \geq 0$

$3 \geq \sqrt[4]{x-1}$

$81 \geq x-1 \to 82 \geq x$

$D_f = [1, 82]$

۳

$f(x) = \dfrac{\log_4 (x^2-x-2)}{\sqrt{x^2-1}+1}$

۰ نمی‌شود

$(x-2)(x+1) > 0$

-1		2	
$+$	$-$		$+$

$\to (-\infty, -1) \cup (2, +\infty)$ ①

$x^2 - 1 \geq 0 \quad x^2 \geq 1$

$x \geq 1$
$x \leq -1$ ②

① $\cap$ ② $= (-\infty, -1) \cup (2, +\infty)$

۴

$f(x) = \sqrt{3 + ax - x^2}$

$D_f = [-2, b]$

$a + b = ?$

$-x^2 + ax + 3 \geq 0$

$-(-2)^2 + a(-2) + 3 = 0$

$-4 - 2a + 3 = 0 \to -1 = 2a \to a = -\frac{1}{2}$

$-x^2 - \frac{1}{2}x + 3 \geq 0$

$\Delta = \frac{1}{4} - 4(-1)(3) = \frac{49}{4}$

$\frac{\frac{1}{2} \pm \sqrt{\Delta}}{-2}$ $\quad \frac{7}{2}$ $\quad \to -2,\ \frac{3}{2}$

-2		$\frac{3}{2}$	
$-$	$+$		$-$

$(-\frac{1}{2}) + (\frac{3}{2}) = \frac{2}{2} = 1$

۵

$f(x) = \begin{cases} 3x-2 & ; x \geq 1 \\ 2x+3 & ; x < 1 \end{cases}$

$g(x) = \sqrt{f(x) - x} \quad f(x) - x \geq 0 \quad f(x) \geq x$

برای اعداد بزرگتر مساوی ۱ همواره برقرار است

و برای اعداد کوچکتر از ۱ از منفی ۳ به بعد برقرار است

$\to [-3, +\infty)$

$x < 1 \to \sqrt{2x+3-x} = \sqrt{x+3} \quad x+3 \geq 0$

$\boxed{x \geq -3}$

$$f(x) = \begin{cases} (a+1)(x+2) & ; x>1 \\ 3a+2x & ; x \le 1 \end{cases}$$

$2f(5) = f(-2) + a$

$a = ?$

$$2(a+1)(7) = 3a + 2(-2) + a$$
$$14a + 14 = 4a - 4$$
$$10a = -18$$
$$\boxed{a = \frac{-18}{10}}$$

6

$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 =$

$f(2-\sqrt{3}) + f(2+\sqrt{3}) = ?$

$$\sqrt{2-\sqrt{3}} + \underbrace{\sqrt{\frac{1}{2-\sqrt{3}}}}_{\sqrt{2+\sqrt{3}}} + 2 + \sqrt{2+\sqrt{3}} + \underbrace{\sqrt{\frac{1}{2+\sqrt{3}}}}_{\sqrt{2-\sqrt{3}}} + 2 = 4 + 2\sqrt{2-\sqrt{3}} + 2\sqrt{2+\sqrt{3}}$$
$$= 4 + \sqrt{2}\left(\sqrt{4-2\sqrt{3}} + \sqrt{4+2\sqrt{3}}\right) =$$
$$4 + \sqrt{2}\left(\sqrt{3}-1+\sqrt{3}+1\right) = 2\sqrt{6} + 4$$

7

$2f(x) - 3f(-x) = 4x^2 - x$

$f(x) = ?$

$$2 \times \quad 2f(x) - 3f(-x) = 4x^2 - x \quad \leftarrow x$$
$$3 \times \quad 2f(-x) - 3f(x) = 4x^2 + x \quad \leftarrow -x$$

$$4f(x) - \cancel{6f(-x)} = 8x^2 - 2x$$
$$\cancel{6f(-x)} - 9f(x) = 12x^2 + 3x$$
$$-5f(x) = 20x^2 + x \rightarrow f(x) = -4x^2 - \frac{1}{5}x$$

8

$(x+2)f(x) - 3x f(x+2) = 4x^2 - mx + 3m - 1$

$f(0) = ?$

$$x = 0 \rightarrow (2)f(0) = 3m - 1 \rightarrow f(0) = \frac{3m-1}{2}$$
$$x = -2 \rightarrow (6)f(0) = 16 + 5m - 1 \rightarrow f(0) = \frac{15+5m}{6}$$

$$\frac{3m-1}{2} = \frac{15+5m}{6} \qquad 9m - 3 = 15 + 5m$$
$$m = \frac{18}{4} = \frac{9}{2} \qquad f(0) = \frac{3\frac{18}{4} - 1}{2} = \frac{50}{8} = \frac{25}{4}$$

9

اگر f تابعی خطی باشد ، $f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 3}{x}$ ، مقدار $f(-1)$ کدام است؟

$$ax + b + a\frac{1}{x} + b = a\left(\frac{1}{x} + x\right) + b$$
$$\frac{3x^2}{x} - \frac{12x}{x} + \frac{3}{x} = 3x - 12 + \frac{3}{x} = 3\left(\frac{1}{x} + x\right) - 12$$

10

$$f(-1) = 3(-1) - 6 = -9$$

$$\boxed{\begin{matrix} a = 3 \\ b = -6 \end{matrix}}$$