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حسابان شماره ۱ — یازدهم ریاضی B — علاءالدینی

۱- الف) $\frac{x-1}{x}-\frac{x}{x-1}\geq 0 \Rightarrow \frac{x^2-2x+1-x^2}{x(x-1)}\geq 0 \Rightarrow \frac{1-2x}{x(x-1)}\geq 0$ $\quad \frac{1}{2}$

$\Rightarrow$

x		0		$\frac{1}{2}$		1	
	$+$	∄	$-$	0	$+$	∄	$-$

$\Rightarrow D=(-\infty,0)\cup[\frac{1}{2},1)$

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ب) $\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}} \Rightarrow$ ① $x\neq 0$ ② $x\neq -1$ ③ $x\neq 1$ ④ $x\neq -2$ ⑤ $\frac{3}{x-1}+\frac{1}{x+2}\neq 0$

$\Rightarrow \frac{3x+6+x-1}{(x-1)(x+2)}=\frac{4x+5}{(x-1)(x+2)}\neq 0 \rightarrow x\neq -\frac{5}{4} \Rightarrow D=\mathbb{R}-\{-2,-\frac{5}{4},-1,0,1\}$

۲- الف) $((\frac{1}{3})^x-9)(32-4^x)\geq 0 \Rightarrow (3^{-x}-3^2)(2^5-2^{2x})\geq 0 \Rightarrow$ $x=-2$, $x=\frac{5}{2}$

~~$-\frac{5}{2}$ -2 ; $- \,∄\, + \,∄\, -$~~

-2 $\quad 2.5$; $+ \;|\; - \;|\; +$

$D=[-\frac{5}{2},-2]$

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ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3 \Rightarrow \sqrt{y+1}=3-\sqrt[4]{x-1}$ $\quad$ ① $x-1\geq 0 \Rightarrow x\geq 1$

② $3-\sqrt[4]{x-1}\geq 0 \Rightarrow \sqrt[4]{x-1}\leq 3 \Rightarrow x-1\leq 81 \quad x\leq 82 \quad D=[1,82]$

۳- $\dfrac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+1} \Rightarrow$ ① $x^2-x-2>0 \Rightarrow (x-2)(x+1)>0$ $\quad$ $-1\quad 2$; $+\,∄\,-\,∄\,+$ $\Rightarrow (-\infty,-1)\cup(2,+\infty)$

② $x^2-1\geq 0 \Rightarrow$ $-1\quad 1$; $+\,0\,-\,0\,+$ $\Rightarrow (-\infty,-1]\cup[1,+\infty)$

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③ $\sqrt{x^2-1}\neq -1$ ✓ $\Rightarrow D=(-\infty,-1)\cup(2,+\infty)$

۴- $D=[-2,b] \Rightarrow 3+ax-x^2=-(x^2-ax-3)=(x+2)(x-b)$

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$\Rightarrow x^2-ax-3=(x+2)(x+b)=x^2+(2+b)x+2b \Rightarrow 2b=-3 \quad b=\pm\frac{3}{2}$

$2+b=-a \Rightarrow a=-(-\frac{3}{2}+2)=-\frac{1}{2} \qquad a+b=-\frac{3}{2}-\frac{1}{2}=-2$

$a+b=1$

۵- $f(x)-x\geq 0$

$\nearrow$ $f(x)\geq x \Rightarrow$ if $x\geq 1 \longrightarrow 3x-2-x\geq 0 \Rightarrow 2x-2\geq 0 \quad x\geq 1$ ✓

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$\searrow$ if $x<1 \longrightarrow 2x+3-x\geq 0 \Rightarrow x\geq -3 \Rightarrow D=[-3,1)\cup[1,+\infty)$

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$$f(a) = (a+1)\times 7 = 7a+7 \quad f(2) = 3a + (-4) \Bigg\} \Rightarrow 2(7a+7) = 3a-4+a$$ ۶
$$14a+14 = 4a-4 \Rightarrow 10a = -18 \quad a = -1.8$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3}) = \sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 4$$ ۷
$$= \sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} + \frac{\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}}{\underbrace{(\sqrt{2-\sqrt{3}})(\sqrt{2+\sqrt{3}})}_{4-3=1}} + 4 \Rightarrow 2(\underbrace{\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}}_{t}) + 4 = A$$
$$\Rightarrow t^2 = (\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}})^2 = 2-\sqrt{3}+2+\sqrt{3}+2 = 6 \Rightarrow t = \sqrt{6}$$
$$A = 2(\sqrt{6}) + 4 = 2\sqrt{6}+4$$

$$x \rightarrow (2f(x) - 3f(-x) = 4x^2 - x)^{\times 2} \Rightarrow 4f(x) - 6f(-x) = 8x^2 - 2x$$ ۸
$$-x \rightarrow (2f(-x) - 3f(x) = 4x^2 + x)^{\times 3} \Rightarrow 6f(-x) - 9f(x) = 12x^2 + 3x$$
$$\overset{+}{\Rightarrow} -5f(x) = 20x^2 + x \Rightarrow f(x) = \frac{-20x^2 - x}{5}$$

$$\xrightarrow{x=0} 2f(0) = 3m-1 \Rightarrow f(0) = \frac{3m-1}{2}$$ ۹
$$\xrightarrow{x=-2} 6f(0) = 16 + 2m + 3m - 1 = 5m+15 \Rightarrow f(0) = \frac{5m+15}{6}$$
$$\frac{3m-1}{2} = \frac{5m+15}{3\times 2} \Rightarrow 9m-3 = 5m+15 \Rightarrow 4m = 18 \rightsquigarrow m = \frac{18}{4}$$
$$\Rightarrow f(0) = \frac{3(\frac{18}{4})-1}{2} = \frac{\frac{54-4}{4}}{2} = \frac{50}{8} = \frac{25}{4}$$

$$f(x) = ax+b \quad f(\tfrac{1}{x}) = \frac{a}{x}+b \Bigg\} \quad f(x)+f(\tfrac{1}{x}) = \frac{ax^2+a+2bx}{x} = \frac{3x^2-12x+3}{x}$$ ۱۰
$$\Rightarrow a = 3 \quad b = \frac{-12}{2} = -6$$
$$f(-1) = 3(-1) - 6 = -9$$