

تکلیف شماره ۲ از مجموعه مسائل ریاضی

$$x \neq 0, x \neq 1$$

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \geq 0$

II) $x^2 - x \neq 0 \Rightarrow x(x-1) \neq 0 \Rightarrow \begin{cases} x \neq 0 \\ x \neq 1 \end{cases}$

مخرج $\frac{(x-1)(x-1)}{(x-1)x} - \frac{x(x)}{x-1(x)} \geq 0 \Rightarrow \frac{x^2 - 2x + 1 - x^2}{x^2 - x} \geq 0 \Rightarrow \frac{-2x + 1}{x^2 - x} \geq 0$

$-2x + 1 = 0 \Rightarrow -2x = -1 \Rightarrow x = \frac{1}{2}$
 $Dy = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x-1} + \frac{1}{x+1}}$

$x+1 \neq 0 \Rightarrow x \neq -1$

مخرج $\frac{(x+1) \cdot 1}{(x+1)(x-1)} + \frac{1(x-1)}{x+1(x-1)} \Rightarrow \frac{x+1+x-1}{x^2-x-1} = \frac{2x}{x^2-x-1} \neq 0$

مخرج $\frac{(x) \cdot 1}{(x)x+1} - \frac{1(x+1)}{x(x+1)} \Rightarrow \frac{x-x-1}{x^2-x} = \frac{-1}{x^2-x}$

الف) $f(x) = \sqrt{(\frac{1}{x}-1)(x-1)} \geq 0$

$x^2 = 1 \Rightarrow x^2 - 1 = 0 \Rightarrow x = 1 \text{ or } x = -1$

$(\frac{1}{x})^2 = 1 \Rightarrow \frac{1}{x^2} = 1 \Rightarrow x^2 = 1 \Rightarrow x = 1 \text{ or } x = -1$

$Dy = (-\infty, -1] \cup [\frac{1}{2}, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 2$

I) $\sqrt{x-1} \rightarrow x-1 \geq 0 \Rightarrow x \geq 1$

II) $\sqrt{y+1} \rightarrow y+1 \geq 0 \Rightarrow y \geq -1$

$Dy = \{(x, y) \in \mathbb{R} \mid x \geq 1, y \geq -1\}$

$f(x) = \frac{\log_5(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$

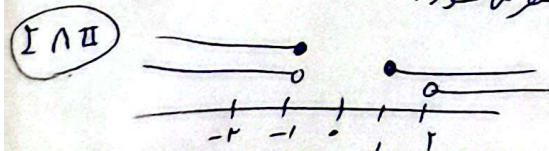
I) $x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0$

$(x < -1) \cup (x > 2)$

II) $x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow (x \geq 1) \cup (x \leq -1)$

III) $\sqrt{x^2 - 1} + 1 \neq 0 \Rightarrow \sqrt{x^2 - 1} \neq -1$

هیچ وقت منفی نشود



$Dy = (-\infty, -1) \cup (1, +\infty)$

$$\sqrt{r+ax-x^2} \quad r+ax-x^2 \geq 0 \quad x = -\frac{b \pm \sqrt{\Delta}}{2a} \quad x = \frac{-a \pm \sqrt{a^2+1r}}{1} \quad (f)$$

$$a+b=1 \quad r+ax-x^2=0$$

$$x=-r \rightarrow r+a(-r)-(-r)^2=0 \Rightarrow a(-r)-1=0 \rightarrow a=-\frac{1}{r}$$

$$b = \frac{a+\sqrt{1r+ar}}{r} = \frac{\sqrt{1r+\frac{1}{2}} - \frac{1}{r}}{r} = \frac{\sqrt{\frac{2r+1}{2}} - \frac{1}{r}}{r} = \frac{r}{r} = 1$$

$$a+b = \frac{r}{r} - \frac{1}{r} = 1$$

$$f(x) = \begin{cases} rx-1 & ; x \geq 1 \\ rx+r & ; x < 1 \end{cases}$$

$$g(x) = \sqrt{f(x)-x} \geq 0 \Rightarrow f(x) \geq x$$

$$f(x)-x = (rx+r)-x = x+r \geq 0 \Rightarrow x \geq -r \Rightarrow -r \leq x < 1$$

$$f(x) = rx-1 \rightarrow f(x)-x = (rx-1)-x = rx-1 \geq 0 \Rightarrow x \geq 1$$

$$Dg = [-r, 1) \cup [1, +\infty) = [-r, +\infty)$$

$$f(x) = \begin{cases} (a+1)(x+r) & ; x > 1 \\ ra+rx & ; x \leq 1 \end{cases}$$

$$rf(a) = f(-r)+a \quad a=9 \quad (4)$$

$$\textcircled{I} f(a) = (a+1)(a+r) = (a+1)v = va+v$$

$$rf(a) = r(va+v) = 12a+14$$

$$rf(a) = f(-r)+a \Rightarrow$$

$$12a+12 = ra - r + a \Rightarrow$$

$$\textcircled{II} f(-r) = ra+r(-r) = ra-r$$

$$12a - ra = -r - 12 \rightarrow a = \frac{-11}{1} = -11$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + r$$

$$f(1-\sqrt{r}) + f(1+\sqrt{r}) = 9$$

$$\sqrt{1-\sqrt{r}} + \frac{1}{\sqrt{1-\sqrt{r}}} + r \rightarrow \sqrt{1-\sqrt{r}} + \sqrt{1+\sqrt{r}} + r$$

$$\sqrt{1+\sqrt{r}} + \frac{1}{\sqrt{1+\sqrt{r}}} + r \rightarrow \sqrt{1+\sqrt{r}} + \sqrt{1-\sqrt{r}} + r$$

$$= 2 + r(\sqrt{1-\sqrt{r}} + \sqrt{1+\sqrt{r}})$$

$$\sqrt{1-\sqrt{r}} + \sqrt{1+\sqrt{r}} + r + \sqrt{1+\sqrt{r}} + \sqrt{1-\sqrt{r}} + r = 2 + 2\sqrt{1-\sqrt{r}} + 2\sqrt{1+\sqrt{r}} + 2r$$

$$2r = 2 + 2\sqrt{1-\sqrt{r}} + 2\sqrt{1+\sqrt{r}} + 2r \rightarrow 2 = 2\sqrt{1-\sqrt{r}} + 2\sqrt{1+\sqrt{r}}$$

$$2 = 2(\sqrt{1-\sqrt{r}} + \sqrt{1+\sqrt{r}})$$

$$rf(x) - rf(-x) = \varepsilon x^r - x$$

$$f(x) = ?$$

(1)

$$\begin{aligned} rf(x) - rf(-x) &= \varepsilon x^r - x \\ rx - rf(x) + rf(-x) &= -rx^r + x \end{aligned} \Rightarrow \begin{cases} \varepsilon f(x) - rf(-x) = rx^r - rx \\ -rf(x) + rf(-x) = -rx^r + rx \end{cases}$$

(110)

$$-2f(x) = \varepsilon x^r + x \Rightarrow f(x) = \frac{-rx^r + x}{-2} =$$

$$\frac{x(-rx+1)}{-2} = \boxed{\frac{r}{2}x^r - \frac{1}{2}x}$$

$$(x+r)f(x) - rf(x+r) = rx^r - mx + rm - 1$$

$$f(0) = ?$$

(9)

$$x=0 \rightarrow rf(0) = rm - 1$$

$$x=-r \rightarrow rf(0) = 14 + rm + rm - 1 \Rightarrow \begin{cases} rf(0) = rm - 1 \rightarrow f(0) = \frac{rm-1}{r} \\ rf(0) = 2m+1 \rightarrow f(0) = \frac{2m+1}{r} \end{cases}$$

$$\frac{rm-1}{r} = \frac{2m+1}{r} \Rightarrow rm-1 = 2m+1 \Rightarrow rm = 2m+2 \Rightarrow m = 1 \Rightarrow m = \frac{1}{\varepsilon} = \frac{1}{r}$$

$$\frac{rm-1}{r} \rightarrow \frac{r \times \frac{1}{r} - 1}{r} = \frac{r-1}{r} \rightarrow \varepsilon$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{rx^r - rx + r}{x}$$

(10)

$$f(-1) = ?$$

(5)

$$\textcircled{I} f(x) + f\left(\frac{1}{x}\right) = ax + b + \frac{a}{x} + b = \left(\frac{1}{x} + x\right)a + 2b$$

$$\frac{rx^r - rx + r}{x} = rx - 1r + \frac{r}{x}$$

$$f(x) = rx - 4 \rightarrow f(-1) = r(-1) - 4$$

$$\left(\frac{1}{x} + x\right)a + 2b = rx - 1r + \frac{r}{x}$$

$$2b = -1r \Rightarrow \boxed{b = -4}$$

$$f(-1) = \boxed{-9}$$