

نام و نام خانوادگی: ... پاسخنامه تشریحی تکلیف شماره ... کلاس: ...

الف) $f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1}$ $\frac{(x-1)^2 - x^2}{x(x-1)} \geq 0$ $\frac{x^2+1-2x-x^2}{x(x-1)} \geq 0$ $\frac{-2x+1}{x(x-1)} \geq 0$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{1}{x-1} + \frac{1}{x+1}}$ $\frac{x \neq -1}{x \neq 0} \frac{x \neq 1}{x \neq -2} \frac{-1}{x-1} + \frac{1}{x+1} \neq 0$ $\frac{x}{x-1} \neq \frac{1}{x-2}$

$-2x-4 \neq x-1$ $x \neq -\frac{5}{3}$ $D_f = \mathbb{R} - \{-2, -\frac{5}{3}, -1, 0, 1\}$

الف) $f(x) = \sqrt{(\frac{1}{x}-1)(x-2)}$ $(\frac{1}{x}-1)(x-2) \geq 0$ $\frac{-1}{x} + \frac{2}{x} \geq 0$ $\frac{1}{x} \geq 0$

$D_f = (-\infty, 2] \cup [1, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 4$ $x-1 \geq 0$ (I) $\sqrt{y+1} = 4 - \sqrt{x-1}$ $-4 + \sqrt{x-1} \geq 0$ $x-1 \leq 1$ (II) $x \leq 2$

$D_f = [1, 2]$

$f(x) = \frac{\log_e(x^2-2-x)}{\sqrt{x^2-1}+1}$ $\frac{x^2-x-2}{(x-1)(x+1)} \geq 0$ $\frac{-1}{x-1} + \frac{2}{x+1} \geq 0$ (I)

$x^2-1 \geq 0$ $\frac{-1}{x-1} + \frac{2}{x+1} \geq 0$ (II)

$\sqrt{x^2-1}+1 \neq 0$ $\sqrt{x^2-1} \neq -1$ $x^2-1 \geq 0$ (III)

$D_f = (-\infty, -1) \cup (1, +\infty)$

$y = \sqrt{x+2} - x$ $D = [-2, b]$

$-x+3-2a=0$

$-2a \leq 1$ $a \geq -\frac{1}{2}$

$-x - \frac{x}{2} + 3 \geq 0$

$\frac{c}{a} = -3$ $\frac{a}{-2} \leq \frac{3}{2}$ $a \geq -3$

$\frac{-2}{-1} + \frac{3}{1} = 1$

$a+b \leq \frac{a}{2} - \frac{1}{2} \leq 1$

$f(x) = \begin{cases} cx-2 & x \geq 1 \\ x+3 & x < 1 \end{cases}$ $g(x) = \sqrt{f(x)} - x$

$f(x) - x \geq 0$ $\frac{-2}{x} + \frac{1}{x} \geq 0$ $D_g = [-3, +\infty)$

$cx-2 = x$ $x=2$ $x \leq 1$

$x+3 = x$ $x = -3$

$$f(x) = \begin{cases} (a+1)(x+1) & x > 1 \\ x+a+1 & x \leq 1 \end{cases}$$

$$f(x) = (a+1)(x+1) \quad \forall x > 1 \quad f(-1) = x+a-1$$

$$f(x) = f(-1) + a$$

$$x+a+1 = x+a-1 \quad a = \frac{-1-1}{1} = \frac{-2}{1} = -2$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 1 \quad f(1-\sqrt{x}) = \sqrt{1-\sqrt{x}} + \frac{1}{\sqrt{1-\sqrt{x}}} + 1$$

$$f(1+\sqrt{x}) = \sqrt{1+\sqrt{x}} + \frac{1}{\sqrt{1+\sqrt{x}}} + 1$$

$$f(1-\sqrt{x}) + f(1+\sqrt{x}) = \sqrt{1-\sqrt{x}} + \frac{1}{\sqrt{1-\sqrt{x}}} + \sqrt{1+\sqrt{x}} + \frac{1}{\sqrt{1+\sqrt{x}}} + 2$$

$$= \sqrt{\frac{(1-\sqrt{x})^2}{1-\sqrt{x}}} + \sqrt{\frac{1}{1-\sqrt{x}}} + 1 + \sqrt{\frac{(1+\sqrt{x})^2}{1+\sqrt{x}}} + \sqrt{\frac{1}{1+\sqrt{x}}} + 1$$

$$= \sqrt{\frac{1-\sqrt{x}}{1-\sqrt{x}}} + \sqrt{\frac{1}{1-\sqrt{x}}} + 1 + \sqrt{\frac{1+\sqrt{x}}{1+\sqrt{x}}} + \sqrt{\frac{1}{1+\sqrt{x}}} + 1$$

$$= 1 + \frac{\sqrt{x}}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{x}} + 2 = 1 + \sqrt{x} + \frac{\sqrt{x}}{x} = 1 + \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$f(x) - f(-x) = x^2 - x$$

$$f(-x) - f(x) = x^2 + x$$

$$f(x) - f(-x) = x^2 - x$$

$$f(-x) - f(x) = x^2 + x$$

$$-2f(x) = x^2 + x \quad \rightarrow \quad f(x) = -\frac{x^2 + x}{2}$$

$$(x+1)f(x) - (x+1)f(x+1) = x^2 - mx + m - 1$$

$$x=0 \rightarrow 1f(0) = 0 - 0 + m - 1 \quad \Rightarrow \quad 4f(0) = 4m - 4$$

$$x=1 \rightarrow 4f(0) = 4 + 2m + m - 1 = 3 + 3m$$

$$4m - 4 = 3 + 3m \quad m = 7$$

$$f(0) = m - 1 = 6 \quad f(0) = \frac{4}{3}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{x^2 - 12x + 1}{x}$$

$$x=-1 \rightarrow f(-1) + f(-1) = \frac{1 + 12 + 1}{-1} = -14 \quad f(-1) = -7$$