آردیس اکبری

یازدهم تجربی B — تکلیف شماره ۴ — ۲۰

سوال ۱)

الف) $\sqrt{\frac{x-1}{x}-\frac{x}{x-1}} \to \begin{cases} x\neq 0 \\ x\neq 1 \end{cases} (I)$

$\frac{x-1}{x}-\frac{x}{x-1}\geq 0 \to \frac{(x-1)^2-x^2}{x(x-1)}\geq 0 \to \frac{1-2x}{x(x-1)}\geq 0$

x		0		$\frac{1}{2}$		1	
	$+$		$-$		$+$		$-$

$D: (-\infty,0)\cup[\frac{1}{2},1)\ (II)$

$\boxed{I\cap II \Rightarrow D_f = (-\infty,0)\cup[\frac{1}{2},1)}$

ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}} \to \begin{cases} x\neq -1,0,1,-2\ (I) \\ \frac{3}{x-1}+\frac{1}{x+2}\neq 0 \to \frac{4x+5}{(x+2)(x-1)}\neq 0 \to x\neq -2,1,-\frac{5}{4}\ (II)\end{cases}$

$\boxed{I\cap II \Rightarrow D_f = \mathbb{R}-\{-2,-\frac{5}{4},-1,0,1\}}$

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سوال ۲)

الف) $f(x)=\sqrt{((\frac{1}{3})^x-9)(32-4^x)} \to ((\frac{1}{3})^x-9)(32-4^x)\geq 0$

x		-2		$\frac{5}{2}$	
	$+$		$-$		$+$

$\boxed{D_f = (-\infty,-2]\cup[\frac{5}{2},+\infty)}$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3 \to \begin{cases} x-1\geq 0 \to x\geq 1\ (I) \\ \underbrace{\sqrt{y+1}}_{\text{نامنفی}} = \underbrace{3-\sqrt[4]{x-1}}_{\text{نامنفی}} \to \sqrt[4]{x-1}\leq 3 \to x-1\leq 81 \to x\leq 82\ (II)\end{cases}$

$\boxed{I\cap II \Rightarrow D_f = [1,82]}$

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سوال ۳)

$f(x)=\dfrac{\log_2(x^2-x-2)}{\sqrt{x^2-1}+1} \to \begin{cases} x^2-x-2>0 \to (x-2)(x+1)>0 \to D=(-\infty,-1)\cup(2,+\infty)\ (I) \\ x^2-1\geq 0 \to D=(-\infty,-1]\cup[1,+\infty)\ (II) \\ \sqrt{x^2-1}\neq -1 \to \text{همیشه برقرار}\end{cases}$

x		-1		2	
	$+$		$-$		$+$

x		-1		1	
	$+$		$-$		$+$

$\boxed{D_f = (-\infty,-1)\cup(2,+\infty)}$

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سوال ۴)

$\sqrt{3+ax-x^2}\ ,\ [-2,b] \xrightarrow{x=-2} 3-2a-4=0 \to -2a-1=0 \to 2a=-1 \to \boxed{a=-\frac{1}{2}}$

$\to -x^2-\frac{1}{2}x+3 \to \Delta=\frac{1}{4}-4(-1)(3)=\frac{49}{4} \to \dfrac{\frac{1}{2}\pm\sqrt{\frac{49}{4}}}{-2} \Rightarrow x=-2,\ \boxed{x=\frac{3}{2}=b}$

$\boxed{a+b=-\frac{1}{2}+\frac{3}{2}=1}$

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سوال ۵)

$f(x)=\begin{cases} 3x-2 & ;\ x\geq 1\ (1) \\ 2x+3 & ;\ x<1\ (2)\end{cases} \qquad g(x)=\sqrt{f(x)-x}$

$(1): 3x-2-x=2x-2\geq 0,\ x\geq 1 \to D=[1,+\infty)\ (I)$

$(2): 2x+3-x=x+3\geq 0 \to x\geq -3 \xrightarrow[(2)]{\text{با توجه به شرط}} -3\leq x<1 \to D=[-3,1)\ (II)$

$\boxed{I\cup II \Rightarrow D_f=[-3,+\infty)}$

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سوال ۶)

$f(x)=\begin{cases}(a+1)(ax+2) & ;\ x>1 \\ 3a+2x & ;\ x\leq 1\end{cases}$

$2f(2)=f(-2)+a$

$2(a+1)(2a+2)=3a+2(-2)+a$

$14a+14=4a-4$

$10a=-18$

$\boxed{a=\frac{-18}{10}=\frac{-9}{5}}$

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$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$ $\quad f(2-\sqrt{3}) + f(\underbrace{2+\sqrt{3}}_{t})$ $\quad ; \quad \frac{1}{2+\sqrt{3}} = 2-\sqrt{3}$

$\sqrt{a} + \frac{1}{\sqrt{a}} + 2$

$\Rightarrow f(t) + f(\frac{1}{t}) = 2(\sqrt{t} + \frac{1}{\sqrt{t}}) + 4 \rightarrow 2(\sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}}) + 4$

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$2(\underbrace{\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}}_{a}) + 4$

$a^2 = (2-\sqrt{3}) + (2+\sqrt{3}) + 2\sqrt{(2+\sqrt{3})(2-\sqrt{3})} \rightarrow a^2 = 6 \quad a = \sqrt{6}$

$f(2+\sqrt{3}) + f(2-\sqrt{3}) = 2a + 4 = \boxed{2\sqrt{6} + 4}$

سوال ۸

$\times 2 \{ 2f(x) - 3f(-x) = 4x^2 - x$

$\times 3 \{ 2f(-x) - 3f(x) = \underbrace{4(-x)^2 - (-x)}_{4x^2+x}$

$\Rightarrow \begin{cases} 4f(x) - 6f(-x) = 8x^2 - 2x \\ -9f(x) + 6f(-x) = 12x^2 + 3x \end{cases}$

$-5f(x) = 20x^2 + x$

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$\boxed{f(x) = \frac{-(20x^2 + x)}{5} = -4x^2 - \frac{x}{5}}$

سوال ۹

$(x+2)f(x) - 3x f(x+2) = 4x^2 - mx + 3m - 1$

$\xrightarrow{x=0} 2f(0) = 3m - 1 \quad \times -3$

$\xrightarrow{x=-2} 6f(0) = 16 + 2m + 3m - 1 \rightarrow 6f(0) = 5m + 15$

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$\rightarrow \begin{array}{l} -6f(0) = -9m + 3 \\ 6f(0) = 5m + 15 \end{array}$

$-4m + 18 = 0 \rightarrow 4m = 18 \rightarrow m = \frac{9}{2}$

$2f(0) = 3(\frac{9}{2}) - 1 = \frac{25}{2} \rightarrow \boxed{f(0) = \frac{25}{4}}$

سوال ۱۰

$f(x) + f(\frac{1}{x}) = \frac{3x^2 - 12x + 3}{x}$

$\xrightarrow{x=-1} f(-1) + f(\frac{1}{-1}) = \frac{3(-1)^2 - 12(-1) + 3}{-1}$

$2f(-1) = -18 \rightarrow \boxed{f(-1) = -9}$

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