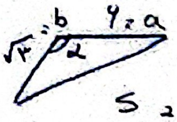
الیاعلی نجاری یازدهم ریاضی B تکلیف ۲۷

-۱



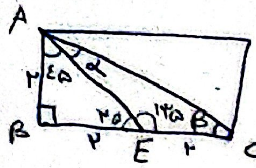
$S = 4{,}5 \qquad \dfrac{\max\alpha}{\min\alpha} = ?$

$S = \frac{1}{2} a \times b \times \sin\alpha = 4{,}5 = \frac{1}{2} \times 6 \times \sqrt{3} \times \sin\alpha \Rightarrow \sin\alpha = \dfrac{4{,}5 \times 2}{6 \times \sqrt{3}} = \dfrac{3}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \alpha = 120^\circ$ یا $60^\circ \longrightarrow \dfrac{\max\alpha}{\min\alpha} = \dfrac{120}{60} = \boxed{2}$

---

-۲



$\cot\alpha ?$

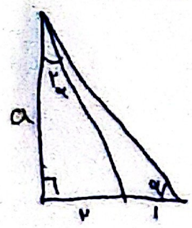
$\tan\beta = \frac{1}{2}$

در $A\hat{E}C \rightarrow \beta + \alpha = 45^\circ \rightarrow \alpha = 45 - \beta \Rightarrow \cot(\alpha) = \dfrac{1}{\tan\alpha} = \dfrac{1}{\tan(45-\beta)}$

$= \dfrac{1 + \tan 45 \tan\beta}{\tan 45 + \tan\beta} = \dfrac{1 + \frac{1}{2}}{1 - \frac{1}{2}} = \dfrac{\frac{3}{2}}{\frac{1}{2}} = \boxed{3}$

---

-۳



$\cot(2\alpha) = \dfrac{a}{3} \Longrightarrow \cot(2\alpha) = \dfrac{1 - \tan^2\alpha}{2\tan\alpha} = \dfrac{1 - \frac{a^2}{9}}{\frac{2a}{3}} = \dfrac{\frac{9-a^2}{9}}{\frac{2a}{3}} = \dfrac{9-a^2}{6a}$

$\cot(\alpha) = \dfrac{3}{a} \rightarrow \tan\alpha = \dfrac{a}{3}$

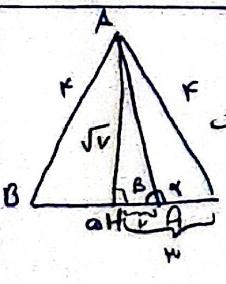
$\Longrightarrow \dfrac{a}{3} = \dfrac{9-a^2}{6a} \rightarrow 3a^2 = 9 - a^2 \rightarrow 4a^2 = 9 \rightarrow a = \pm\dfrac{3}{2}$

غ ق ق

$\cot\alpha = \dfrac{3}{\frac{3}{2}} = \boxed{2}$

---

-۴



$\tan\alpha ?$

فیثاغورث: $AH^2 = AC^2 - HC^2 = 16 - 9 = 7 \rightarrow AH = \sqrt{7}$

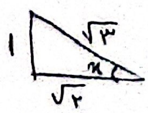
$\alpha = 180 - \beta \Longrightarrow \tan\alpha = \tan(180 - \beta) = -\tan\beta = -\dfrac{AH}{HD} = \boxed{-\dfrac{\sqrt{7}}{2}}$

---

-۵

$2\sin^2 x + \cos^2 x = \dfrac{4}{3} \qquad \tan^2 x = ?$

$\sin^2 x + \underbrace{\sin^2 x + \cos^2 x}_{1} = \dfrac{4}{3} \Longrightarrow \sin^2 x = \dfrac{1}{3} \rightarrow \sin x = \pm\dfrac{1}{\sqrt{3}}$



$\rightarrow \tan x = \pm\dfrac{1}{\sqrt{2}} \rightarrow \boxed{\tan^2 x = \dfrac{1}{2}}$

---

-۶

$\dfrac{\sin^4\alpha + 4\cos^2\alpha}{1 + \cos^2\alpha} - \dfrac{\cos^4\alpha + 4\sin^2\alpha}{1 + \sin^2\alpha} = \dfrac{\sin^4\alpha + 4\cos^2\alpha + \sin^6\alpha + \cancel{4\cos^2\alpha\sin^2\alpha} - \cos^4\alpha - 4\sin^2\alpha - \cos^6\alpha - \cancel{4\sin^2\alpha\cos^2\alpha}}{1 + \underbrace{\sin^2\alpha + \cos^2\alpha}_{1} + \sin^2\alpha\cos^2\alpha}$

$(\sin^4\alpha - \cos^4\alpha + \sin^6\alpha - \cos^6\alpha + 4\cos^2\alpha - 4\sin^2\alpha) \div (2 + \sin^2\alpha\cos^2\alpha)$

$(\sin^2\alpha - \cos^2\alpha)(\underbrace{\sin^2\alpha + \cos^2\alpha}_{1}) \qquad (\sin^2\alpha - \cos^2\alpha)(\sin^4\alpha + \cos^4\alpha + \sin^2\alpha\cos^2\alpha)$

$3\cos^2\alpha - 3\sin^2\alpha = 3(\cos^2\alpha - \sin^2\alpha) \qquad (\underbrace{\sin^2\alpha + \cos^2\alpha}_{1})^2 - 2\sin\alpha\cos\alpha$

$= \sin^2\alpha - \cos^2\alpha + 4\cos^2\alpha - 4\sin^2\alpha + (\cos^2\alpha - \sin^2\alpha)(\sin\alpha\cos\alpha - 1) \div (2 + \sin^2\alpha\cos^2\alpha)$

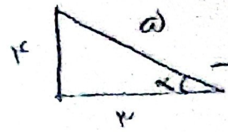
$= (\cos^2\alpha - \sin^2\alpha)(\cancel{\sin\alpha\cos\alpha + 2}) \div (\cancel{\sin^2\alpha + \cos^2\alpha + 1}) = \cos^2\alpha - \sin^2\alpha = \boxed{\cos(2\alpha)}$

۷-

$\tan\alpha = \frac{4}{3}$

$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) - \tan\left(\alpha - \frac{3\pi}{2}\right)$

$= \sin\left(4\pi + \frac{\pi}{2} + \alpha\right)\cos\left(2\pi + \frac{3\pi}{2} - \alpha\right) + \tan\left(\frac{3\pi}{2} - \alpha\right)$



$\sin\alpha = -\frac{4}{5}$

$\cos\alpha = -\frac{3}{5}$

$= -\cos(\alpha)\sin(\alpha) + \cot(\alpha) = \underbrace{-\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right)}_{-\frac{12}{25}} + \left(\frac{3}{4}\right) = \frac{-48+75}{100} = \boxed{0/27}$

---

۸-

$x = \frac{\pi}{12}$

$3\cos 4x + \underbrace{\sqrt{2}\sin x - \sqrt{2}\cos x}_{\sqrt{2}(\sin x - \cos x)} = 3\cos 4x + 2\sin\left(x - \frac{\pi}{4}\right) \xrightarrow{x = \frac{\pi}{12}} 3\cos\left(\frac{\pi}{3}\right) + 2\sin\left(\frac{\pi}{12} - \frac{3\pi}{12}\right)$

$\sqrt{2}\,(\underbrace{\sin x - \cos x}_{\sqrt{2}\sin\left(x - \frac{\pi}{4}\right)})$

$= 3\cos\left(\frac{\pi}{3}\right) - 2\sin\left(\frac{\pi}{6}\right)$

$= 3 \times \frac{1}{2} - 1 = \boxed{\frac{1}{2}}$

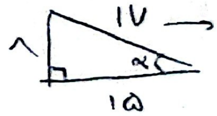
---

۹-

$\tan\left(\frac{\alpha}{2}\right) = \frac{1}{4}$

$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha}$

$\tan\alpha = \frac{2\tan\left(\frac{\alpha}{2}\right)}{1-\tan^2\left(\frac{\alpha}{2}\right)} = \frac{2\times\frac{1}{4}}{1-\frac{1}{16}} = \frac{160}{300} = \frac{8}{15}$ ⟶



17 ← فیثاغورث $\sqrt{15^2+8^2}$

$\sin\alpha = \frac{8}{17}$

$\cos\alpha = \frac{15}{17}$

$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{136-120}{255}}{\frac{136-255}{289}} = \frac{\frac{16}{255}}{\frac{-119}{289}} = \boxed{\frac{-16}{105}}$

---

۱۰-

انتهای کمان $\alpha$ در کدام ربع؟

$2\sin\alpha < \sin 2\alpha$

$2\sin\alpha < 2\sin\alpha\cos\alpha$

$\sin\alpha < \sin\alpha\cos\alpha$

اگر $\sin\alpha > 0$ ↓ $1 < \cos\alpha$ غ ق ق

اگر $\sin\alpha < 0$ ↓ $1 > \cos\alpha$ ✓ ق ق ⟶ $\sin\alpha < 0$

$\frac{\cot\alpha}{\sin\alpha} > 0$ ↓ $\frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha} > 0$ ↓ $\frac{\cos\alpha}{\sin^2\alpha} > 0 \Rightarrow \cos\alpha > 0$

$\Rightarrow$ ربع چهارم