Subject:
Year: ۱۴۰۲ Month: ۱۲ Day: ۲۳

بسمه تعالی

فاطمه متقی - ۲۷
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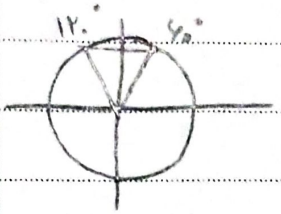
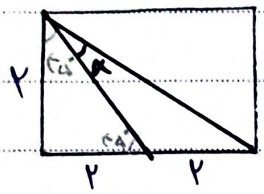
۱) $S = \frac{1}{2}a = \frac{1}{2} \times 4 \times \sqrt{3} \times \sin\alpha \rightarrow \sin\alpha = \frac{2a}{4\sqrt{3}} = \frac{3\sqrt{3}}{2\times 3} = \frac{\sqrt{3}}{2}$



$Max\alpha = 120°$

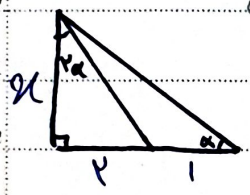
$min\alpha = 60°$

۲) $\cot(45+\alpha) = \frac{1}{2} = \frac{1 - \tan 45 \cdot \tan\alpha}{\tan 45 + \tan\alpha} = \frac{1-\tan\alpha}{1+\tan\alpha}$



طرفین وسطین $\rightarrow 2 - 2\tan\alpha = 1 + \tan\alpha \Rightarrow 3\tan\alpha = 1 \rightarrow \cot\alpha = 3$

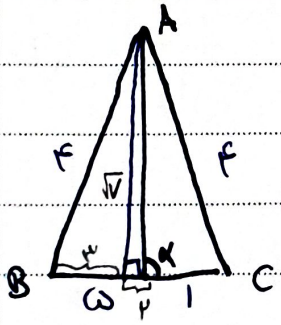
۳) $\cot 2\alpha = \frac{x}{2}$, $\cot\alpha = \frac{3}{x}$, $\cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}$



$\frac{x}{2} = \frac{\frac{9-x^2}{x^2}}{\frac{6}{x}} \Rightarrow \frac{x}{2} = \frac{9-x^2}{6x} \rightarrow 3x^2 = 9 - x^2 \rightarrow x^2 = \frac{9}{4}$

$x = \frac{3}{2} \longrightarrow \cot\alpha = 2$

۴) ارتفاع و میانه در مثلث متساوی الساقین یکسان است.



$h_a = \sqrt{7} \rightarrow \tan(180 - \alpha) = -\tan\alpha$

$\hookrightarrow -\tan\alpha = \frac{\sqrt{7}}{2} \Rightarrow \tan\alpha = \frac{-\sqrt{7}}{2}$

۵) $2\sin^2 x + \cos^2 x = \frac{4}{3} \rightarrow \sin^2 x + \cancel{\sin^2 x + \cos^2 x}^{1} = \frac{4}{3}$

$\sin^2 x = \frac{1}{3} \rightarrow \cot^2 x = \frac{1}{\sin^2 x} - 1 \Rightarrow \cot^2 x = 2 \rightarrow \tan^2 x = \frac{1}{2}$

۶) $\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} \Rightarrow \frac{(1-\cos^2\alpha)^2 + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{(1-\sin^2\alpha)^2 + 4\sin^2\alpha}{1+\sin^2\alpha}$

$\hookrightarrow \frac{\cos^4\alpha + 2\cos^2\alpha + 1}{1+\cos^2\alpha} - \frac{\sin^4 x + 2\sin^2 x + 1}{1+\sin^2\alpha} \rightarrow \frac{(1+\cos^2\alpha)^2}{1+\cos^2\alpha} - \frac{(1+\sin^2\alpha)^2}{(1+\sin^2\alpha)}$

$1 + \cos^2\alpha - 1 - \sin^2\alpha = \cos 2\alpha$

Arman

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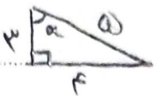
$\sin\left(\frac{۹\pi}{۲}+\alpha\right)\cos\left(\frac{۷\pi}{۲}-\alpha\right)-\tan\left(\alpha-\frac{۳\pi}{۲}\right) \rightarrow \tan\left(\frac{۳\pi}{۲}-\alpha\right)$

$\cos\alpha \times (-\sin\alpha)+\cot\alpha$



$\longrightarrow \frac{۳}{۵}\times\frac{-۴}{۵}+\frac{۳}{۴} = \frac{-۴۸+۷۵}{۱۰۰} = ۰/۲۷$

(۸

$۳\cos ۴\alpha+\sqrt{۲}\sin\alpha-\sqrt{۲}\cos\alpha \Rightarrow ۳\cos\frac{\pi}{۳}+\sqrt{۲}(\sin\alpha-\cos\alpha)$

$\frac{۳}{۲}+\sqrt{۲}\left(\sqrt{۲}\sin\left(\alpha-\frac{\pi}{۴}\right)\right)=\frac{۳}{۲}+۲\sin\left(\frac{\pi}{۱۲}-\frac{\pi}{۴}\right)=\frac{۳}{۲}-۲\sin\frac{\pi}{۶}=\frac{۳}{۲}-۱=\frac{۱}{۲}$

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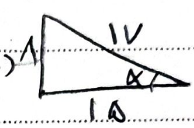
$\tan\alpha=\frac{۲\tan\left(\frac{\alpha}{۲}\right)}{۱-\tan^{۲}\left(\frac{\alpha}{۲}\right)} \Rightarrow \tan\alpha=\frac{\frac{۱}{۲}}{\frac{۱۵}{۱۶}}=\frac{۸}{۱۵} \Rightarrow$



$\cos\alpha=\frac{۱۵}{۱۷}$

$\sin\alpha=\frac{۸}{۱۷}$

$\frac{\frac{۸}{۱۵}-\frac{۸}{۱۷}}{\frac{۸}{۱۷}-\frac{۱۵}{۱۷}}=\frac{\frac{۸(۱۷-۱۵)}{۱۵\times۱۷}}{\frac{۸-۱۵}{۱۷}}=\frac{-۱۶}{۱۰۵}$

(۱۰

① $\frac{\cos\alpha}{\sin\alpha}\times\frac{۱}{\sin\alpha}=\frac{\cos\alpha}{\sin^{۲}\alpha}>۰ \Rightarrow \cos\alpha>۰$

② $۲\sin\alpha<۲\sin\alpha\cos\alpha \Rightarrow \sin\alpha\underbrace{(۱-\cos\alpha)}_{\text{همواره نامنفی}}<۰ \Rightarrow \sin\alpha<۰$

① و ② $\Rightarrow \cos\alpha>۰$ و $\sin\alpha<۰ \longrightarrow$

$\longrightarrow$ ناحیه چهارم