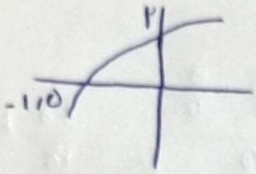
۱-



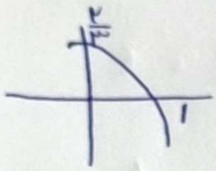

$y = 1 - \log_c^{ax-b}$ , $b+c = \frac{-۳}{۲}$ → $\begin{vmatrix}۰\\۲\end{vmatrix}$ $\begin{vmatrix}-\frac{۳}{۲}\\۰\end{vmatrix}$

$۲ = 1 + \log_c^{-b}$ → $b = \frac{-1}{c}$ , $c - \frac{1}{c} = \frac{-۳}{۲}$ → $c = \frac{1}{۲}$ ✓ , $-۲$ غ ق ق , $۰<c<1$

$c = \frac{1}{۲}$ , $b = -۲$ , $۰ = 1 - \log_c^{\frac{-۳a}{۲} - b}$ → $c+b = \frac{-۳}{۲}$ → $a = 1$

$(a+c)b = \frac{۳}{۲} \times -۲ = \boxed{-۳}$

۲-



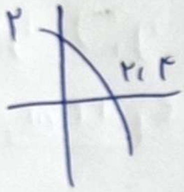

$f(x) = 1 + c \times ۳^{a+bx}$ $\begin{vmatrix}1\\۰\end{vmatrix}$ $\begin{vmatrix}۰\\\frac{۲}{۳}\end{vmatrix}$

$۰ = 1 + c \times ۳^{a+b}$ → $۳^{a+b} = \frac{-1}{c}$ → $\log_۳^{-c^{-1}} = a+b$ → $-\frac{1}{c} > ۰$

$\frac{۲}{۳} = 1 + c \times ۳^{a}$ → $-۳^{-1} = c \times ۳^{a}$ → $c = -1$ , $a = -1$ → $b+a = ۰$ , $b = 1$

$f(-1) = 1 + -1 \times ۳^{-1-1} = 1 - ۳^{-۲}$ → $1 - \frac{1}{۹} = \boxed{\frac{۸}{۹}}$

۳-




$y = c + \log_۵^{ax+b}$ $\begin{vmatrix}۲٫۴\\۰\end{vmatrix}$ $\begin{vmatrix}۰\\۲\end{vmatrix}$ → $-c = \log_۵^{۲٫۴a+b}$ → $۵^{-c} = -۰٫۱a$

$۲ = c + \log_۵^{b}$ → $۵^{-c} = \frac{b}{۲۵}$

$\frac{b}{۲۵} = \frac{-1a}{1۰}$ → $\frac{a}{b} = \frac{-1۰}{۲۵} = -۰٫۴$

۴-

$f(x) = \log_۴^{|x^۲-۲|-x}$ → $|x^۲-۲| - x > ۰$ → $|x^۲-۲| > x$ → $x^۲ - ۲ > x$

① $(x-۲)(x+1) > ۰$ $\quad -1 \quad ۲ \quad + \; - \; +$

$x^۲ - ۲ < -x$

② $(x+۲)(x-1) < ۰$ $\quad -۲ \quad 1 \quad + \; - \; +$

① ∪ ② $= D_f = (-\infty, 1) \cup (۲, \infty)$

۵-

$f(x) = ۲ + ۲^{b-ax}$ $\quad g(x) = -x^۲ - ۳x + ۸$ $\quad x = 1$ $\quad f^{-1}(1۰) = -1$

$g(1) = f(1)$ → $۴ = ۲ + ۲^{b-a}$ → $۲^{b-a} = ۲$ → $b - a = 1$

$f(-1) = 1۰$ → $1۰ = ۲ + ۲^{a+b}$ → $a + b = ۳$ ⇒ $b = ۲$ , $a = 1$

$۲b - a = ۴ - 1 = \boxed{۳}$

۶-

$f(x) = -۲ + (\frac{1}{۲})^{Ax+B}$ $\quad y = x^۲ - x$ $\quad x = 1, ۲$

$f(1) = g(1)$ → $۰ = -۲ + ۲^{-A-B}$ → $-A-B = 1$ , $f(۲) = g(۲)$ → $۲ = -۲ + ۲^{-۲A-B}$ → $۲A+B = -۲$

⇒ $A = -1$ , $B = ۰$ , $f(۳) = -۲ + ۲^{۳} = \boxed{۶}$

۷-

$\lg_2^5 = 2.8 \quad \lg_3^5 = 1.4$ ، $t =$ ساعات ، $M =$ جرم اولیه $\rightarrow \frac{M}{6} = M\left(\frac{8}{9}\right)^t \rightarrow \lg_{\frac{8}{9}}^{\frac{1}{6}} = t \rightarrow$

$t = -\lg_{\frac{8}{9}}^{6} \xrightarrow{\text{تغییر مبنا}} \frac{\lg_5^6}{\lg_5^{\frac{8}{9}}} = \frac{\lg_5^2 + \lg_5^3}{\lg_5^8 - \lg_5^9} = \frac{\frac{5}{14} + \frac{5}{7}}{3\lg_5^2 - 2\lg_5^3} = -\frac{\frac{5}{14} + \frac{5}{7}}{\frac{15}{14} - \frac{10}{7}} = \frac{-95}{-15}h =$

$\frac{19}{3}h \times 60\,\text{min} = \underline{380\,\text{min}}$

۸-

$\lg_2^3 = 1.6 \quad \lg_7^3 = 0.6$ ، $t =$ هفته ، $M =$ جرم اولیه $\Rightarrow \frac{M}{7} = M\left(\frac{7}{8}\right)^t \rightarrow \lg_{\frac{7}{8}}^{\frac{1}{7}} = t$

$-\lg_{\frac{7}{8}}^{7} = t \rightarrow t = \xrightarrow{\text{تغییر مبنا}} \frac{-\lg_3^7}{\lg_3^{\frac{7}{8}}} = \frac{-\lg_3^7}{\lg_3^7 - 3\lg_3^2} = \frac{\frac{10}{6}}{\frac{10}{6} - \frac{15}{8}} = 8$ هفته

$\underline{8 \times 7 = 56}$ روز

۹- در هر مرحله غلظت $\frac{24}{25}$ خواهد شد

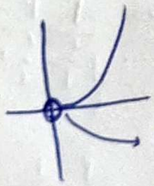
$\lg 2 = 0.3 \quad \lg 3 = 0.48 \quad \frac{100}{100}\left(\frac{24}{25}\right)^t = \frac{1}{3} \times \frac{100}{100} \rightarrow$

$\left(\frac{96}{100}\right)^t = \frac{1}{3} \rightarrow (t)\lg\frac{96}{100} = \lg\frac{1}{3} \rightarrow (t)(\lg 96 - \lg_{2}^{100}) = -\lg 3$

$(t)(5\lg 2 + \lg 3 - 2) = -\lg 3 \rightarrow (t)(1.5 + \frac{48}{100} - 2) = -0.48 \rightarrow t = \underline{24}$ روز

۱۰-

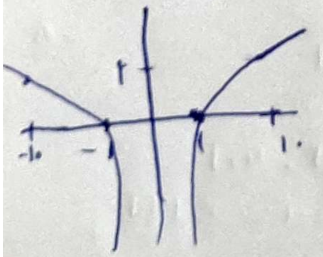
الف) $y = 9^{\lg_3^x} \xrightarrow{\text{ساده}} = x^{\lg_3^9} = x^2 \rightarrow x > 0 \rightarrow$  $\Rightarrow$



(از صفر توخالی)

$\rightarrow$ جواب

ب) $y = \lg x^2 \rightarrow x = \mathbb{R} - \{0\} \rightarrow$ چون $x^2 > 0 \quad y = 0 \rightarrow x = 1$




$\rightarrow$ جواب