

$$\log_n^m = a \quad \& \quad \log_{mn}^{m^2 n} = b \rightarrow \frac{\log_n^{m^2 n}}{\log_n^{m+1}} = \frac{b - \log_n^{m+1}}{\log_n^{m+1}} \rightarrow \frac{b+1}{a+1}$$

$$\frac{r(a+1)-1}{a+1} = r - \frac{1}{a+1} \rightarrow 1 < r - \frac{1}{a+1} < r \rightarrow [b] = 1$$

$$\text{ب) } \sqrt{\frac{x}{\log \frac{1}{x}}} \rightarrow x > 0 \rightarrow \log \frac{x}{\frac{1}{x}} \neq 0 \rightarrow x \neq 1 \quad \text{و} \rightarrow x = e \rightarrow \log \frac{e}{\frac{1}{e}} = -2$$

$$\text{ج) } \log x^2 - x - 2 \sim (x-1)(x+1) \sim \frac{-1}{x+1} \rightarrow \sqrt{x^2-1} + 1 \rightarrow x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \geq 1 \rightarrow x \leq -1$$

$$r \log_a^a + \log_a^{\sqrt{a}} = r \xrightarrow{x=a} r \log_a^a + \log_a^{\sqrt{a}} = r \quad \log_a^a + \log_a^{\sqrt{a}} = r$$

$$m + \frac{1}{m} = \frac{m^2+1}{m} = r \rightarrow m^2 - r m + 1 = 0 \rightarrow (m-1)^2 \rightarrow m-1 = \log_a^a$$

$$\log \frac{a}{n} = \log a - \log n = \log \frac{1}{n} - \log n = \log \frac{1}{n^2} = -2 \log n$$

$$\log 9 = 2 \log 3 = 2 \log \frac{1}{n} = -2 \log n \quad \log 16 = \log 4 + \log 4 = \log 10 - \log 2 + \log 10 - \log 2 = 6 - 2 \log 2$$

$$\log \frac{1}{n} = -\log n$$

$$\log \frac{1}{n^2} = -2 \log n \rightarrow \log \frac{1}{n} = -\log n$$

$$\log \frac{1}{n} = \frac{\log \frac{1}{n}}{\log \frac{1}{n}} = \frac{\log \frac{1}{n} + \log \frac{1}{n}}{\log \frac{1}{n} + \log \frac{1}{n}} = \frac{2 \log \frac{1}{n}}{2 \log \frac{1}{n}} = 1$$

$$\log_{\frac{1}{w}} \frac{1}{u} = \frac{\log \frac{1}{u}}{\log \frac{1}{w}} = \frac{\log u}{\log w} + 1$$

$$\log_{\frac{1}{u}} \frac{1}{v} = \frac{1}{1} \rightarrow \frac{1}{\log \frac{1}{u}} = \frac{1}{1} \rightarrow \log \frac{1}{u} = 1 \rightarrow \frac{1}{u} = 10^1 = 10$$

$$\frac{1}{10} + \frac{1}{10} = \frac{2}{10} = \frac{1}{5}$$

$$\log_{\frac{1}{n}} \frac{1}{m} = m \rightarrow \frac{1}{m} \log_{\frac{1}{n}} \frac{1}{m} = \frac{1}{m} (\log \frac{1}{m} + m \log \frac{1}{n}) \rightarrow \frac{1}{m} (1 + m \log \frac{1}{n}) = m$$

$$\log_{\frac{1}{n}} \frac{1}{m} = \frac{1}{m} \log \frac{1}{m} = \frac{1}{m} (\log \frac{1}{m} + \log \frac{1}{n})$$

$$m = 1 + m \log \frac{1}{n} \rightarrow \log \frac{1}{n} = \frac{m-1}{m}$$

$$\frac{1}{m} (1 + m \log \frac{1}{n}) = \frac{1}{m} (1 + m \log \frac{1}{n})$$

$$\left(\frac{1}{n}\right)^{m-1} = \left(\frac{1}{n}\right)^{m-1} \rightarrow \frac{1}{n^{m-1}} = \frac{1}{n^{m-1}} \rightarrow n^{m-1} = 1$$

$$m^2 + m - 1 = 0 \rightarrow m^2 + m - 1 = 0 \rightarrow m = \frac{1}{2} (1 \pm \sqrt{5})$$

$$\log_{\frac{1}{n}} \frac{1}{m} + 1 = \log_{\frac{1}{n}} \frac{1}{m}$$

$$\frac{1}{m} \log_{\frac{1}{n}} \frac{1}{m} = \frac{1}{m} (a+1) \Rightarrow \log_{\frac{1}{n}} \frac{1}{m} = a+1 \rightarrow \log_{\frac{1}{n}} \frac{1}{m} = a+1$$

$$\log_{\frac{1}{n}} (m \times m - 1) = m$$

$$5 = \frac{b}{ka} \rightarrow \frac{1}{5} = \frac{ka}{b} \Rightarrow ka = \frac{b+c}{5} \rightarrow ka = \frac{b+c}{5}$$

$$\log_{\frac{1}{ka}} \frac{1}{b} = \frac{1}{ka} \log \frac{1}{b} = \frac{1}{ka} \log \frac{1}{b} \rightarrow \log \frac{1}{b} = \frac{ka}{b}$$

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