آزمایش رحیم‌زاده - یازدهم دختر B - تکلیف ۲۳

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$\log_n m = a \to \log_m n = \frac{1}{a} \Rightarrow \log_m n + \log_m m = \log_m mn = \frac{1}{a} + 1 \to \log_m mn = \frac{a+1}{a} \to \log_{mn} m = \frac{a}{a+1}$

$\log_{mn} m^2 n = \log_{mn} m + \log_{mn} mn = b \to \frac{a}{a+1} + 1 = b \to \left[\frac{a}{a+1}\right] + 1 = 1 = [b]$

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الف) $y = \sqrt{\dfrac{x}{\log_{\frac{1}{2}} x}}$

$\hookrightarrow \log_{\frac{1}{2}} x > 0$ $\quad x > 0$ و $\log_{\frac{1}{2}} x > 0 \to x < 1 \quad \Rightarrow D_y = (0,1)$

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ب) $y = \dfrac{\log_2 (x^2 - x - 2)}{\sqrt{x^2-1} + 1}$ $\quad \to x^2 - x - 2 > 0 \to$ $\begin{array}{ccccc} & -1 & & 2 & \\ + & | & - & | & + \end{array}$

$\sqrt{x^2-1} + 1 \to \sqrt{x^2-1} + 1 \neq 0$ ✓

$D_y = (-\infty, -1) \cup (2, +\infty)$

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$2\log_x a + \log_a \sqrt{x} = 2 \to 2\log_x a + \frac{1}{2\log_x a} - 2 = 0 \to 4(\log_x a)^2 + 1 - 4\log_x a = 0 \to (2\log_x a - 1)^2$

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$2\log_9 a = 1 \to \underline{a = 3}$

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$\log 2 \simeq 0/3$

$\log 3 \simeq 0/4$

$\to \underbrace{(\log_{10} 10 - \log_{10} 2 - \log_{10} 3)}_{1 - 0/3 - 0/4} x^2 + \underbrace{(\log 3 + \log 3)}_{0/4 + 0/4} x - \underbrace{(\log 10 - \log 2 + \log 3)}_{1 - 0/3 + 0/4}$

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$(\log \frac{5}{3}) x^2 + (\log 9) x - \log 15 = 0$

$\hookrightarrow 0/3 x^2 + 0/8 x - 1/1 = 0$

$S = -\frac{8}{3} \quad P = \frac{-11}{3}$

$|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{S^2 - 4P}$

$\sqrt{\frac{64}{9} + \frac{44}{3}} = \sqrt{\frac{196}{9}} = \boxed{\frac{14}{3}}$

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$\log_2 7 = 2/8 \to \log_2 7 + \log_2 2 = 3/8 = \log_2 14$

$\log_5 2 = 0/5 \to \log_2 5 = 2 \to \log_2 5 + \log_2 2 = \log_2 10 = 3$

$\log_{14} 10 = \frac{15}{19}$

$\to \dfrac{\log_2 10}{\log_2 14} = \frac{3}{3/8} = \frac{30}{38} = \frac{15}{19}$

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$\log_3 5 = 1/5 \to \log_3 5 + \log_3 3 = \log_3 15 = 2/5$

$\log_2 3 = 1/6 \to \log_3 2 = \frac{5}{8} \to \log_3 2 + \log_3 3 = \log_3 6 = \frac{13}{8}$

$\log_{15} 6 = ? \to \dfrac{\log_3 6}{\log_3 15} = \dfrac{\frac{13}{8}}{\frac{5}{2}} = \frac{13}{20}$

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$\log_۸^{۱۸} = m \to \frac{\log_۲^{۱۸}}{\log_۲^{۸}} = m \to \log_۲^{۱۸} = ۳m \qquad \log_۲^{۱۸} - \log_۲^{۲} = \log_۲^{۳^۲} = ۲\log_۲^{۳} = ۳m-۱$

$\log_۲^{۳} = \frac{۳m-۱}{۲}$

$\log_۴^{۱۲} = \frac{۳m+۳}{۴}$

$\log_۴^{۱۲} = \frac{\log_۲^{۱۲}}{\log_۲^{۴}} = \frac{\log_۲^{۳} + \log_۲^{۴}}{۲} = \frac{\frac{۳m-۱}{۲}+۲}{۲} = \frac{۳m-۱+۴}{۴} = \frac{۳m+۳}{۴}$

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-۸

$(۰/۴)^{۲x-۱} = \left(\frac{۱۲۵}{۸}\right)^{x^۲} \to \left(\frac{۲}{۵}\right)^{۲x-۱} = \frac{۲}{۵}^{-۳x^۲} \rightsquigarrow ۳x^۲+۲x-۱=۰ \longrightarrow x \begin{cases} -۱ \\ \frac{۱}{۳} \end{cases}$

$\log_۸^{۹x+۱} \longrightarrow ۹x+۱>۰ \to x > -\frac{۱}{۹} \to x = \frac{۱}{۳} \rightsquigarrow \log_۸^{۴} = \log_{۲^۳}^{۲^۲} = \boxed{\frac{۲}{۳}}$

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$\log_۲^{۳} = a \to \log_۲^{۲} + \log_۲^{۳} = \log_۲^{۶} \rightsquigarrow \frac{۲}{۳}(\log_۲^{۶}) = \log_۸^{۳۶}$

$\log_۸^{b} = \frac{۲}{۳}(۱+a) \qquad \to b = ۳۶$

$\log(۳b-۸) \rightsquigarrow \log_{۱۰}^{۱۰۰} = \boxed{۲}$

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-۱۰

$-۲ax^۲+bx+\frac{۱}{۲}c=۰ \longrightarrow \frac{۱}{\alpha+\beta} = \log ۴ \qquad \alpha+\beta = \frac{b}{۲a} \to \frac{۱}{\alpha+\beta} = \frac{۲a}{b} = \log ۴ \to \frac{b}{a} = ۲\log_۴^{۱۰}$

$\left(\frac{۱}{\sqrt[۸]{۲}}\right)^{\frac{c}{a}} = \left(۲^{-\frac{۱}{۸}}\right)^{\frac{c}{a}} \qquad \frac{b+c}{۲} = a \to ۲a = b+c$

$\hookrightarrow c = b-۲a$

$\frac{c}{a} = \frac{۲a-b}{a} = ۲-\frac{b}{a} = ۲(۱-۲\log_۴^{۱۰}) = ۲(۱-\log_۲^{۱۰})$

$۱.۲\log_۲^{\frac{۲}{۱۰}}$

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$\left(\left(\frac{۱}{۲}\right)^{۸}\right)^{۲\log_۲^{\frac{۲}{۱۰}}} = \left(\left(\frac{۱}{۲}\right)^{\frac{۲}{۱۰}}\right)^{۲\log_۲^{۸}} = \left(\left(\frac{۱}{۲}\right)^{\frac{۲}{۱۰}}\right)^{۶} = ۲^{-\frac{۱۲}{۱۰}}$

$$a=\frac{b+c}{2} \rightarrow b=2a-c \rightarrow \frac{1}{x_1+x_2}=\frac{1}{S}=\frac{4a}{b}=\frac{4a}{2a-c}=2\log^{2}$$

$$\rightarrow \frac{2a}{2a-c}=\log^{2} \xrightarrow{\text{عکس}} \frac{2a-c}{2a}=\log_{2}^{10} \rightarrow 1-\log_{2}^{10}=\frac{c}{2a} \rightarrow 2-\log_{2}^{100}=\frac{c}{a} \quad (1)$$

$$\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}}=\left(2^{-\frac{1}{8}}\right)^{\frac{c}{a}}=\left(2^{\frac{c}{a}}\right)^{-\frac{1}{8}} \xrightarrow{(1)} \left(2^{2-\log_{2}^{100}}\right)^{-\frac{1}{8}}$$

$$\left(\frac{2^{2}}{2^{\log_{2}^{100}}}\right)^{-\frac{1}{8}}=\left(\frac{4}{100}\right)^{-\frac{1}{8}}=(25)^{\frac{1}{8}}=\sqrt[4]{5}$$