♡ ۱۳ سارا جعفری ♡

①

$\log_n^m = a$ ، $\log_{mn}^{m^2 n} = b$ ، $a>0$ $0<\frac{a}{a+1}<1 \to 1<b<2$ $\boxed{[b]=1}$

$\frac{\log m}{\log n} = a \to \log m = a\log n$ $\frac{\log m^2 n}{\log mn} = \frac{2\log m+\log n}{\log m+\log n} = \frac{2a\log n+\log n}{a\log n+\log n} \to b = \frac{2a+1}{a+1}$ $1+\frac{a}{a+1}$

②

الف) $y=\sqrt{\frac{x}{\log_{\frac{1}{2}} x}}$ I) $x>0$ II) $\log_{\frac12} x \neq 0 \to x\neq 1$ III $= \frac{x}{\log_{\frac12} x} \geq 0$

$I\cap II\cap III = \boxed{(0,1)}$

ب) $y = \frac{\log_2 (x^2-x-2)}{\sqrt{x^2-1}+1} = I\cap II: (-\infty,-1)\cup(2,+\infty)$

I: $x^2-1\geq 0 \to |x|\geq 1 \to x\in(-\infty,-1]\cup[1,+\infty)$

II: $x^2-x-2>0 \to x=-1/2$ $x\in(-\infty,-1)\cup(2,+\infty)$

③

$2\log_x^a + \log_a^{3x} = 2 \xrightarrow{x=9} 2\log_{9}^{a} + \log_a^3 = 2 \to \log_3^a + \log_a^3 = 2 \to t=1 \to \underline{a=3}$

④

$\log 2 = 0/3$ $\log 3 = 0/4$ $\log 5 = 0/7$

$(\log\frac{5}{3})x^2 + \log 9\, x - \log 15 = 0$

$(\log 5-\log 3)x^2 + 2\log 3\, x - (\log 3+\log 5) \Rightarrow 0/3x^2 + 0/8x + 1/1 = 0$

$x_2 = \frac{-1/1}{0/3}$ ، $x_1 = 1 \to Dx = \frac{14}{3}$

⑤

$\log_2^7 = 2/8$ ، $\log_5^3 = 0/5$ $\log_{14}^{10} = ?$

$\frac{\log_2^{14}}{\log_2^{10}} \Rightarrow \frac{\log_2^7+\log_2^2}{\log_2^5+\log_2^2} \Rightarrow \frac{2/8+1}{3} \to \frac{3/8}{3}$ عکس $\log_{14}^{10} = \frac{3}{3/8}$

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$\log 2 = 1/6\ ,\ \log_3^5 = 1/5 \qquad \log_{15}^6 = ?$

$\log_3^2 = \frac{5}{8}$

$$\frac{\log_3^6}{\log_3^{15}} = \frac{\log_3^2 + \log_3^3}{\log_3^5 + \log_3^3} = \frac{\frac58 + 1}{\underset{2/5}{1/5+1}} \to \frac{\frac{13}{8}}{\frac{5}{2}} \to \frac{26}{40} \to \frac{13}{20}$$

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$\log_8^{18} = m \qquad \log_4^{12} = ? \to \frac{\log_8^{12}}{\log_8^4} \Rightarrow \frac{\log_8^4 + \log_8^3}{\log_{2^3}^{2^2}} \to \frac{\frac23 + \log_8^3}{\frac23} = \frac{\frac23 + \frac{m-\frac13}{2}}{\frac23}$

$\log_8^9 + \log_8^2$

$2\log_8^3 + \underset{\frac13}{\log_{2^3}^{2}} = m \quad \to \frac{m-\frac13}{2} = \log_8^3$

$$\frac{\frac{4+3m-1}{6}}{\frac23} \to \frac{12+9m-3}{12}$$

$$\boxed{\frac34(m+1)} \leftarrow \frac{9+9m}{12}$$

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$(0/4)^{2n-1} = \left(\frac{125}{8}\right)^{n^2} \qquad \log_8^{9n+1} = ? \qquad \log_8^4 \to \log_{2^3}^{2^2} \to \frac23$

$\left(\frac25\right)^{2n-1} = (2/5)^{3n^2} \to (2/5)^{1-2n} = (2/5)^{3n^2} \to 3n^2 = 1-2n \to 3n^2+2n-1 = 0$

$\boxed{n = -1,\ \frac13}$

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$\log_2^3 = a\ ,\ \log_8^b = \frac23(1+a) \qquad \log^{3b-8} = ? \qquad 2^a = 3$

$\log_2^b = 2(1+a) \to b = 2^{2(1+a)} = 2^{2+2a} = 2^2 \times 2^{2a} \Rightarrow 4 \times (2^a)^2$

$b = 4 \times 3^2 = 36$

$\log^{3b-8} = \log^{100} = 2$

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$\frac{-1}{\frac{b}{-2a}} = \log^4 = \frac{2a}{b} = \log^4 \quad \frac ab = \frac{\log^4}{4} = \frac{2\log^2}{4} = \frac{\log^2}{2} \qquad 2a = b + c$

$\frac{2a}{b} = 1 + \frac cb$

$\frac cb = -1 + \log^2 \qquad \frac cb = \log^2 - 1 \qquad 2\left(\frac{\log^2}{2}\right) = 1 + \frac cb$

$\frac ab = \frac{\log^2}{2} \to \frac ca = \frac{\log^2 - 1}{\frac{\log^2}{2}}$

$\frac{\log\frac15}{\frac12\log^2} = \log_{\sqrt2}^{\frac15} \to \left(\frac{1}{\sqrt2}\right)\log_{\sqrt2}^{\frac15} \to \left(\frac15\right)\log_{\sqrt2}^{\sqrt2} \to 25$