نام و نام خانوادگی: رزا حمیدی شماره تکلیف: ۲۳ کلاس: سه‌شنبه‌ها گروه B دهم تا یازدهم

1

$\log_n m = a \quad \log_{mn} m^2 n = b$

$[b] \quad \log_{mn} m^2n = \dfrac{\log_n m^2 n}{\log_n mn} = \dfrac{\log_n n + 2\log_n m}{\log_n n + \log_n m} = \dfrac{1+2a}{1+a}$

$\rightarrow [b] = \left[\dfrac{1+2a}{1+a}\right] = \left[1 + \dfrac{a}{1+a}\right] = 1 + \left[\dfrac{a}{1+a}\right] = \boxed{1}$

2

الف) $y = \sqrt{\dfrac{x}{\log_{\frac{1}{2}} x}}$

$\log_{\frac{1}{2}} x \neq 0 \rightarrow x \neq 1$ ①

$\dfrac{x}{\log_{\frac{1}{2}} x} \geq 0 \xrightarrow{x>0} \log_{\frac{1}{2}} x > 0 \rightarrow x < 1$ ②

③ $x > 0$

③ $\cap$ ① $\cap$ ② $= D = \boxed{(0, 1)}$

ب) $y = \dfrac{\log_4 (x^2 - x - 2)}{\underbrace{\sqrt{x^2-1} + 1}_{\text{همواره مثبت}}} \rightarrow D = (-\infty, -1) \cup (2, +\infty)$

$x^2 - x - 2 > 0$

$(x-2)(x+1) > 0$

-1		2
$+$	$-$	$+$

صورت $D = (-\infty, -1) \cup (2, +\infty)$

مخرج D $\quad x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow (-\infty, -1] [1, +\infty)$

3

$2\log_x a + \log_a \sqrt{x} = 2 \quad , x = 9$

$\rightarrow 2\log_x a + \dfrac{1}{\log_{\sqrt{x}} a} = 2 \rightarrow 2\log_x a + \dfrac{1}{2\log_x a} = 2 \rightarrow 2t + \dfrac{1}{2t} = 2 \rightarrow 4t^2 - 4t + 1 = 0$

$\rightarrow (2t-1)^2 = 0 \rightarrow 2t = 1 \rightarrow t = \dfrac{1}{2} \rightarrow \log_9 a = \dfrac{1}{2} \rightarrow a = 9^{\frac{1}{2}} \rightarrow a = \sqrt{9}$

$\rightarrow \boxed{a = 3}$

4

$\log 2 = 0/3 \, , \log 3 = 0/4 \rightarrow \log 2 = \log 10 - \log 5 = 0/3 \rightarrow 0/7 = \log 5$

$(\log \frac{5}{3})x^2 + (\log 9)x - \log 15 = 0$

$(\log 5 - \log 3)x^2 + (2\log 3)x - (\log 3 + \log 5) = (0/7 - 0/4)x^2 + (2 \times 0/4)x - (0/7 + 0/4)$

$= (0/3)x^2 + (0/8)x - 1/1 = 0 \rightarrow 3x^2 + 8x - 11 = 0 \rightarrow \Delta = 64 + 4 \times 3 \times 11 = 196$

$\rightarrow \dfrac{-8 \pm 14}{6} \Rightarrow \left\{1, -\dfrac{11}{3}\right\} \leftarrow x$

$\boxed{1 + \dfrac{11}{3} = \dfrac{14}{3}}$

5

$\log_2 7 = 2/8 \; و \; \log_5 2 = 0/5 = \dfrac{1}{\log_2 5} = \dfrac{5}{10} = \dfrac{1}{2} \rightarrow \log_2 5 = 2$

$\log_{14} 10 = \dfrac{\log_2 10}{\log_2 14} = \dfrac{\log_2 2 + \log_2 5}{\log_2 7 + \log_2 2} = \dfrac{1+2}{2/8 + 1} = \dfrac{3}{3/8} = \boxed{\dfrac{30}{38}}$

$\log_3^5 = 1.5 \quad , \log_2^3 = 1.6 \qquad \log_3^5 = \frac{\log_2^5}{\log_2^3} = 1.5 \to \log_2^5 = 2.4$

$\log_{15}^6 = \frac{\log_2^6}{\log_2^{15}} = \frac{\log_2^2 + \log_2^3}{\log_2^3 + \log_2^5} = \frac{1+1.6}{1.6+2.4} = \frac{2.6}{4} = 0.65$ 6

$\log_8^{18} = \frac{1}{3}(\log_2^9 + \log_2^2) = m \to 2\log_2^3 + 1 = 3m \to \log_2^3 = \frac{3m-1}{2}$

$\log_4^{12} = \frac{1}{2}\log_2^{12} = \frac{1}{2}(\log_2^3 + \log_2^4) = \frac{1}{2}\left(\frac{3m-1}{2} + 2\right) = \frac{3m-1}{4} + 1$

$\left(\frac{4}{10}\right)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \to \left(\frac{2}{5}\right)^{2x-1} = \left(\frac{2}{5}\right)^{-3x^2} \to 2x-1 = -3x^2$

$\to 3x^2 + 2x - 1 = 0 \to -1 \ , \ \frac{1}{3}$

$\log_8^{9x+1} \to x=-1 \to \log_8^{-8}$ ✗

$\to x = \frac{1}{3} \to \log_8^4 = \frac{2}{3}$ 8

$\log_2^3 = a$

$2^a = 3$

$\log_8^b$

$b = 8^{\frac{2}{3}a} \times 8^{\frac{2}{3}} = 2^{2a} \times 2^2 = 3^2 \times 2^2 = 36 = b$

$\log_{10}^{100} = 2$ 9

$S = x_1 + x_2 \to \frac{1}{S} = \frac{1}{x_1+x_2} = 2\log^{[illegible]} \to \frac{1}{\frac{b}{4a}} = \frac{4a}{\frac{\cancel{b}}{2a-c}} = 2\log^{[illegible]} \to \frac{2a-c}{2a} = \log_2^{10}$

$1 - \frac{c}{2a} = \log_2^{10} \to \frac{c}{2a} = 1 - \log_2^{10} \xrightarrow{\times 2} \frac{c}{a} = 2 - \log_2^{100}$

$\left(\frac{1}{8\sqrt{2}}\right)^{\frac{c}{a}} = \left(\frac{1}{8\sqrt{2}}\right)^{2-\log_2^{100}} = \left(2^{\frac{1}{8}}\right)^{-2+\log_2^{100}} \longrightarrow \left(\frac{2^2}{2^{\log_2^{100}}}\right)^{-\frac{1}{8}} = \left(\frac{4}{100}\right)^{-\frac{1}{8}}$ 10

$\to \left(\frac{100}{4}\right)^{+\frac{1}{8}} \to (25)^{\frac{1}{8}} \to (5^2)^{\frac{1}{8}} \to 5^{\frac{1}{4}} = \sqrt[4]{5}$