نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۳ ..... کلاس ..... B .....

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$\log_n^m = a \rightarrow \frac{\log m}{\log n} = a \qquad b = \log_{m^2 n}^{mn} = \frac{2\log m + \log n}{\log m + \log n} = \frac{2a+1}{a+1} =$

$1 + \frac{a}{a+1} \rightarrow [b] = 1$

$0 < \frac{a}{a+1} < 1$

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$x^2 - x - 2 > 0 \qquad x < -1 \text{ یا } x > 2$ ①

$x^2 - 1 \geq 0 \qquad x^2 \geq 1 \qquad x \geq 1 \text{ یا } x \leq -1$ ②

$\sqrt{x^2-1} + 1 \neq 0$ ③

① ∧ ② ∧ ③ $(-\infty, -1) \cup (2, +\infty)$

$x > 0 \rightarrow \frac{x}{\log x} \geq 0 \rightarrow \log_{\frac{1}{2}}^{x} > 0$ ①

$\rightarrow 0 < x < 1$ ②

① ∧ ② : $0 < x < 1$

۳

$x = 9 \rightarrow 2\log_9^a + \log_a^3 = 2 \rightarrow \log_3^a + \log_a^3 = 2 \xrightarrow{\log_3^a = t} t + \frac{1}{t} = 2$

$\rightarrow t^2 - 2t + 1 = 0 \rightarrow t = 1 \rightarrow \log_3^a = 1 \rightarrow a = 3$

۴

$\log\left(\frac{5}{2}\right) = \log 5 - \log 2 = \log\frac{10}{2} - \log 2 = \log 10 - \log 2 - \log 2 = 1 - 0{,}3 - 0{,}3 = 0{,}4$

$\log 9 = 2\log 3 = 0{,}8 \qquad \log 15 = \log 5^{\times 3} = \log\frac{10}{2} + \log 3 = \log 10 - \log 2 + \log 3 = 1 - 0{,}3 + 0{,}4 = 1{,}1$

$0{,}3x^2 + 0{,}8x - 1{,}1 = 0 \rightarrow x = 1 \text{ یا } x = \frac{-1{,}1}{0{,}3} = \frac{-11}{3}$

$a - b = 1 - \left(-\frac{11}{3}\right) = \frac{14}{3}$

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$\log_{\sqrt{2}}^{5} = 0{,}5 \rightarrow \log_2^5 = \frac{1}{\log_5^2} = 2$

$\log_{12}^{10} = \frac{\log_2^{10}}{\log_2^{12}} = \frac{\log_2^2 + \log_2^5}{\log_2^3 + \log_2^2} = \frac{1+2}{1+1{,}8} = \frac{3}{2{,}8} = \frac{30}{28} = \frac{15}{14}$

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$$\log_{15}^{6} = \frac{\log_{2}^{6}}{\log_{2}^{15}} = \frac{\log_{2}^{2}+\log_{2}^{3}}{\log_{2}^{5}+\log_{2}^{3}} = \frac{\frac{a}{3}+1}{\frac{a}{3}+1} = \frac{4a}{100} = 0.4a$$

$$\log_{2}^{3} = b \longrightarrow \log_{3}^{2} = \frac{1}{\log_{2}^{3}} = \frac{1}{b} = \frac{a}{3}$$

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$$\log_{6}^{12} = \frac{\log_{3}^{12}}{\log_{3}^{6}} = \frac{\log_{3}^{4}+\log_{3}^{3}}{\log_{3}^{3}+\log_{3}^{2}} = \frac{2\log_{2}^{2}+\frac{1}{2}\log_{2}^{3}}{\frac{1}{2}\log_{2}^{2}+\frac{1}{2}\log_{2}^{3}} = \frac{2+\frac{1}{2}(2m-1)}{\frac{1}{2}} =$$

$$\frac{3}{4}(m+1)$$

$$\log_{18}^{12} = \log 9 + \log_{3}^{2} = m \longrightarrow \frac{2}{3}\log_{2}^{3} + \frac{1}{2}\log_{2}^{2} = m \longrightarrow \frac{2}{3}\log_{2}^{3} + \frac{1}{2} = m$$

$$\log_{2}^{3} = \frac{2m-1}{3}$$

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$$(0.4)^{2x-1} = \left(\frac{1}{2.5}\right)^{x^2} \longrightarrow \left(\frac{2}{5}\right)^{2x-1} = \left(\left(\frac{2}{5}\right)^{2}\right)^{x^2} \longrightarrow \left(\frac{2}{5}\right)^{-2x+1} = \left(\frac{2}{5}\right)^{2x^2}$$

$$2x^2 = -2x+1 \longrightarrow 2x^2+2x-1=0 \begin{cases} x=-1 \text{ ✗} \\ x=\frac{1}{2} \text{ ✓} \end{cases} \longrightarrow \log_{8}^{9\times\frac{1}{2}+1} = \log_{8}^{\frac{3}{2}} = \frac{2}{3}$$

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$$\log_{8}^{b} = \frac{2}{3}(1+a) \xrightarrow{\log_{2}^{3}=a} \log_{8}^{b} = \frac{2}{3}(1+\log_{2}^{3}) \longrightarrow \frac{1}{3}\log_{2}^{b} = \frac{2}{3}(\log_{2}^{2}+\log_{2}^{3})$$

$$\longrightarrow \frac{1}{3}\log_{2}^{b} = \frac{2}{3}\log_{2}^{6} \longrightarrow \frac{1}{3}\log_{2}^{b} = \frac{1}{3}\log_{2}^{36} \longrightarrow b = 36$$

$$\log^{2b-7}_{10} = \log^{100}_{10} = 2$$

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$$-2ax^2+bx+\frac{1}{2}c=0 \rightsquigarrow S = \frac{-b}{-2a} = \frac{b}{2a} \longrightarrow \frac{2a}{b} = \log 2 \xrightarrow{\div 2} \frac{a}{b} = \frac{1}{2}\log 2 =$$

$$\log\sqrt{2}$$

$$\rightarrow a = b\log\sqrt{2}$$

$$2a = b+c \longrightarrow c = 2a-b = 2b\log\sqrt{2} - b = b(2\log\sqrt{2}-1) = b(\log 2 - 1)$$

$$\rightarrow c = -b\log a$$

$$\left(\frac{1}{\sqrt{2}}\right)^{\frac{c}{a}} = (2^{-\frac{1}{2}})^{\frac{-b\log a}{b\log\sqrt{2}}} = 2^{\frac{1}{2}\log_{2}^{a}} = 2^{\log_{2}^{\sqrt{a}}} =$$

$$\sqrt{a}^{\frac{1}{\log_{2}^{2}}} = \sqrt[4]{a}$$