نام و نام خانوادگی ....[illegible].... پاسخنامه تشریحی تکلیف شماره ..۲۳.. کلاس ..[illegible]..

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$\log_n m = a \to \frac{\log m}{\log n} = a$

$b = \log_{mn} m^2 n = \frac{2\log m + \log n}{\log m + \log n} = \frac{2a+1}{a+1} = 1 + \underbrace{\frac{a}{a+1}}_{0<\frac{a}{a+1}<1} \to \boxed{[b]=1}$

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$x^2 - x - 2 > 0 \quad x<-1$ یا $x>2$ (1)

$x^2 - 1 \geq 0 \quad x^2 \geq 1 \quad x \geq 1$ یا $x \leq -1$ (2)

$\sqrt{x^2-1} + 1 \neq 0$ ✓ (3)

(1) ∩ (2) ∩ (3) $\boxed{(-\infty, -1) \cup (2, +\infty)}$

$\underset{(1)}{x>0} \to \frac{x}{\log_{\frac{1}{2}} x} \geq 0 \to \log_{\frac{1}{2}} x > 0 \to 0<x<1$ (2)

(1) ∩ (2) : $\boxed{0<x<1}$

۳

$x=9 \to 2\log_9 a + \log_a 3 = 2 \to \log_3 a + \log_a 3 = 2 \xrightarrow{\log_3 a = t} t + \frac{1}{t} = 2 \to t^2 - 2t + 1 = 0$

$t=1 \Rightarrow \log_3 a = 1 \to \boxed{a=3}$

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$\log\left(\frac{5}{3}\right) = \log 5 - \log 3 = \log\frac{10}{2} - \log 3 = \log 10 - \log 2 - \log 3 = 1 - 0,3 - 0,4 = 0,3$

$\log 9 = 2\log 3 = 0,8 \qquad \log 15 = \log (5 \times 3) = \log\frac{10}{2} + \log 3 = \log 10 - \log 2 + \log 3 = 1 - 0,3 + 0,4 = 1,1$

$0,3x^2 + 0,8x - 1,1 = 0 \begin{cases} x = 1 \\ x = \frac{-1,1}{0,3} = \frac{-11}{3} \end{cases} \qquad a - b = 1 - \left(-\frac{11}{3}\right) = \boxed{\frac{14}{3}}$

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$\log_5 2 = 0,5 \to \log_2 5 = \frac{1}{\log_5 2} = 2$

$\log_{14} 10 = \frac{\log_2 10}{\log_2 14} = \frac{\log_2 2 + \log_2 5}{\log_2 2 + \log_2 7} = \frac{1+2}{1+2,8} = \frac{3}{3,8} = \frac{30}{38} = \boxed{\frac{15}{19}}$

$$\frac{\log_3^{6}}{\log_3^{15}}=\frac{\log_3^{3}+\log_3^{2}}{\log_3^{5}+\log_3^{3}}=\frac{1+\frac{10}{19}}{1.5+1}=\boxed{\frac{13}{20}}$$

$$\log_2^{3}=\frac{1}{\log_3^{2}}=\frac{10}{19}$$

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$$\log_4^{12}=\frac{\log_8^{12}}{\log_8^{4}}=\frac{\log_8^{4}+\log_8^{3}}{\log_8^{4}}=\frac{\frac{2}{3}\log_2^{2}+\frac{1}{3}\log_2^{3}}{\frac{2}{3}\log_2^{2}}=\frac{\frac{2}{3}+\frac{1}{3}\left(\frac{3m-1}{2}\right)}{\frac{2}{3}}=\frac{3}{4}(m+1)$$

$$\log_8^{18}=\log_8^{9}+\log_8^{2}=m\rightarrow\frac{2}{3}\log_2^{3}+\frac{1}{3}\log_2^{2}=m\rightarrow\frac{2}{3}\log_2^{3}+\frac{1}{3}=m\rightarrow\boxed{\log_2^{3}=\frac{3m-1}{2}}$$

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$$(0.4)^{2x-1}=\left(\frac{125}{8}\right)^{x^2}\rightarrow\left(\frac{2}{5}\right)^{2x-1}=\left(\left(\frac{5}{2}\right)^3\right)^{x^2}\rightarrow\left(\frac{2}{5}\right)^{2x-1}=\left(\frac{5}{2}\right)^{3x^2}\rightarrow\left(\frac{5}{2}\right)^{-2x+1}=\left(\frac{5}{2}\right)^{3x^2}$$

$$3x^2=-2x+1\rightarrow 3x^2+2x-1=0\ \begin{cases}x=-1 \ \times\\ x=\frac{1}{3}\ \checkmark\end{cases}\rightarrow\log_8^{9\times\frac{1}{3}+1}=\log_8^{4}=\boxed{\frac{2}{3}}$$

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$$\log_8^{b}=\frac{2}{3}(1+a)\xrightarrow{\log_2^{3}=a}\log_8^{b}=\frac{2}{3}(1+\log_2^{3})\rightarrow\frac{1}{3}\log_2^{b}=\frac{2}{3}(\log_2^{2}+\log_2^{3})\rightarrow$$

$$\frac{1}{3}\log_2^{b}=\frac{2}{3}\log_2^{6}\rightarrow\frac{1}{3}\log_2^{b}=\frac{1}{3}\log_2^{36}\rightarrow b=36$$

$$\log^{3b-8}\xrightarrow{b=36}\log^{3\times36-8}=\log^{100}=\boxed{2}$$

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$$S=\frac{-b}{-4a}=\frac{b}{4a}\rightarrow\frac{4a}{b}=\log^{2}\xrightarrow{\div4}\frac{a}{b}=\frac{1}{4}\log^{2}=\log^{\sqrt{2}}\rightarrow a=b\log\sqrt{2}$$

$$2a=b+c\rightarrow c=2a-b=2b\log\sqrt{2}-b=b(2\log\sqrt{2}-1)=b(\log^{2}-1)\rightarrow c=-b\log^{5}$$

$$\left(\sqrt[8]{\frac{1}{2}}\right)^{\frac{c}{a}}=\left(2^{-\frac{1}{8}}\right)^{\frac{-b\log5}{b\log\sqrt{2}}}=2^{\frac{1}{4}\log_2^{5}}=2^{\log_2^{\sqrt[4]{5}}}=\sqrt[4]{5}^{\log_4^{2}}=\sqrt[4]{5}$$

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