هلیا غفاری

$\log_n^m = a \to n^a = m \qquad \log_{mn}^{m^2 n} = b \qquad \log_{n^a \times n}^{(n^a)^2 \times n}$

(۱)

$\log_{n^{a+1}}^{n^{2a+1}} = b \to b = \frac{2a+1}{a+1}\log_n^n \to \frac{2a+1}{a+1} = \frac{a+(a+1)}{a+1} = 1 + \frac{a}{a+1}$

$\Rightarrow [b] = \left[1+\frac{a}{a+1}\right] = 1 + \left[\frac{a}{a+1}\right] \to [b] = 1$

$1 > \frac{a}{a+1} > 0 \Leftarrow a > 0$

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(۲)

الف) $\sqrt{\frac{x}{\log_{\frac{1}{2}} x}}$ $\qquad \frac{x}{\log_{\frac{1}{2}} x} \geq 0 \xrightarrow{x=0,1}$ $\quad - \ 0 \ + \ 1 \ -$

$D_f = [0, 1)$

ب) $\log(x^2 - x - 2)$ $\qquad x^2 - x - 2 > 0 \quad (x-2)(x+1) > 0$ $\quad + \ -1 \ - \ 2 \ +$

$\frac{4}{\sqrt{x^2-1}+1}$ ← صفر نمیشه $\qquad x^2 - 1 \geq 0 \quad (x-1)(x+1)$ $\quad + \ -1 \ - \ 1 \ +$

$D_f = \mathbb{R} - [-1, 2]$

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(۳)

$2\log_9^a + \log_a^3 = 2 \to \log_3^a + \log_a^3 = 2$

$if\ a = 3 \to \log_3^3 + \log_3^3 = 2 \to a = 3$

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(۴)

$\left(\log\frac{5}{2}\right)x^2 + (\log 9)x - \log 15$ (a, b, c) $\quad -\log 15 = \log 15^{-1} = \log\frac{1}{15}$

$a + b + c = \frac{5}{2} \times 9 \times \frac{1}{15}$

$\Rightarrow a + b + c = \log 1 = 0$ جمع ضرایب صفر $\to$ ریشه‌ها $= 1, \frac{c}{a} \to \frac{-\log 15}{\log\frac{5}{2}}$

$\log 2 = 0.3 \to \log 2 = \log\frac{10}{1} - \log 5 \to \log 5 = \log 10 - \log 2 = 1 - 0.3 = 0.7$

$\frac{c}{a} = \frac{-(\log 5 + \log 3)}{\frac{\log 5}{\log 2} = \frac{0.7}{0.4} = \frac{7}{4}} = \frac{-1.1}{\frac{7}{4}} = \frac{-\frac{11}{10}}{\frac{7}{4}} = \frac{22}{35}$ $\qquad \frac{\Delta}{\text{الف}} = 1 - \frac{22}{35} = \frac{13}{35}$

(۵)

$$\log_{14}^{10} = \frac{\log_2^{10}}{\log_2^{14}}$$

$\log_2^{10} = \log_2^2 + \log_2^5 \rightarrow \log_2^{10} = 1 + 2 = 3$

$\log_2^{14} = \log_2^2 + \log_2^7 = 1 + 2.8 = 3.8$

$\star \ \log_5^2 = \frac{1}{2} \Rightarrow \log_2^5 = 2$ $\qquad \log_{14}^{10} = \frac{3}{3.8}$

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(۶)

$$\log_{15}^6 = \frac{\log_3^6}{\log_3^5}$$

$\log_3^6 = \log_3^3 + \log_3^2 = 1 + \frac{10}{16} = \frac{26}{16}$

$\log_3^{15} = \log_3^5 + \log_3^3 = 1.5 + 1 = 2.5$

$\star \ \log_3^2 = \frac{1}{\log_2^3} = \frac{10}{16}$

$$\frac{\log_3^6}{\log_3^{15}} = \frac{\frac{26}{16}}{\frac{25}{10}} = \frac{26 \times 10}{16 \times 25} = \frac{65}{100}$$

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(۷)

$\log_8^{18} = m \rightarrow \log_8^2 + \log_8^9 = m \rightarrow \frac{2}{3}\log_2^3 = m - \frac{1}{3}$

$\log_2^3 = \frac{3}{2}\left(m - \frac{1}{3}\right) \qquad \log_4^{12} = \log_4^4 + \log_4^3$

$\Rightarrow \log_4^{12} = 1 + \frac{1}{2}\log_2^3 = 1 + \frac{1}{2} \times \frac{3}{2}\left(m - \frac{1}{3}\right)$

$\Rightarrow \log_4^{12} = 1 + \frac{m}{3} - \frac{1}{9} = \frac{8}{9} + \frac{3m}{9} = \boxed{\frac{8 + 3m}{9}}$

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(۸)

$\left(\frac{4}{10}\right)^{2m-1} = \left(\frac{125}{8}\right)^{x^2} \rightarrow \left(\frac{2}{5}\right)^{2m-1} = \left(\frac{5^3}{2^3}\right)^{x^2} \rightarrow \left(\frac{5}{2}\right)^{1-2x} = \left(\frac{5}{2}\right)^{3x^2}$

$\Rightarrow 3x^2 = 1 - 2x \rightarrow 3x^2 + 2x - 1 = 0 \xrightarrow{a+c=b} x = -1, \ x = \frac{1}{3}$

$\log_8^{(9m+1)} \rightarrow 9m + 1 > 0 \rightarrow m > -\frac{1}{9} \xrightarrow{x=\frac{1}{3}} \log_8^4 = \log_{2^3}^{2^2} = \frac{2}{3}\log_2^2 = \boxed{\frac{2}{3}}$

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(۹)

$\frac{2}{3}(1 + a) = \frac{2}{3}\left(\log_2^2 + \log_2^3\right) = \frac{2}{3}\log_2^6 = \log_{2^3}^{6^2} = \log_8^{36}$

$\Rightarrow \log_8^b = \log_8^{36} \rightarrow b = 36 \qquad \log_{10}^{(3 \times 36 - 8)} = \log_{10}^{100} = \boxed{2}$