نام و نام خانوادگی ...[illegible]... پاسخنامه تشریحی تکلیف شماره ۱ ...... کلاس ...[illegible]...

$\log_n m = a \to \frac{\log m}{\log n} = a$ $b = \log_{mn} m^2 n = \frac{2\log m + \log n}{\log m + \log n} = \frac{2a+1}{a+1} = 1 + \frac{a}{a+1}$ 1/... $[b] = 1$	۱
الف) $y = \sqrt{\frac{x}{\log_{\frac12} x}} \Rightarrow \log_{\frac12} x \neq 0 \Rightarrow \log_{\frac12} x = 0 \to x \neq 1$ $\frac{x}{\log_{\frac12} x} \geq 0 \Rightarrow$ $0<x<1$ مثبت است ، $x>1$ منفی $\Rightarrow (0,1)$ ب) $\frac{\log(x^2-x-2)}{\sqrt{x^2-1}+1} \Rightarrow x^2-x-2>0 \Rightarrow (x-2)(x+1)>0 \Rightarrow x<-1$ یا $x>2$ I II $x \geq 1 \Rightarrow (-\infty, -1) \cup (2, +\infty)$ $\sqrt{x^2-1}+1 \Rightarrow x^2-1 > 0 \Rightarrow x \leq -1$	۲
$\log_y x = \frac{1}{\log_x y}$ $2\log_9 a + \log_a 3 = 2 \Rightarrow \frac{2}{2}\log_3 a + \log_a 3 = 2 \Rightarrow \log_3 a + \log_a 3 = 2 \to \log_3 a + \frac{1}{\log_3 a} = 2 \Rightarrow$ $(\log_3 a)^2 + 1 = 2(\log_3 a)$, $\log_3 a = t$, $t^2+1 = 2t \Rightarrow t^2-2t+1=0 \Rightarrow (t-1)^2 = 0 \Rightarrow t=1$ $\log_3 a = 1 \Rightarrow a = 3$	۳
$\log_{10} 2 = 0.3$, $\log_{10} 3 = 0.48 \to 2(\log 3)x = 0.18$ $(\log_{10} \frac{5}{2})x^2 + (\log 9)x - \underbrace{\log 15}_{\log 5 + \log 3 = \log\frac{10}{2} \Rightarrow \log 10 - \log 2 + \log 3 \Rightarrow \frac{1 - 0.3 + 0.48}{1.1}}$ $\log 5 - \log 2 \Rightarrow \log\frac{10}{4} = \log 10 - \log 2 - \log 2 = 0.4$ $0.4x^2 + 0.18x - 1.1 = 0 \xrightarrow{\times 10} 4x^2 + 8x - 11 = 0 \Rightarrow x^2 + 8x - 33 = 0 \Rightarrow$ $(x+11)(x-3) \Rightarrow x_1 = -\frac{11}{3}$, $x_2 = 1 \Rightarrow x_1 - x_2 = 1 - (\frac{11}{3}) = \frac{14}{3}$	۴
$\log_5 2 = \frac12 \Rightarrow \frac{\log 2}{\log 5} = \frac12 \Rightarrow \frac{\log 2}{\log\frac{10}{2}} = \frac12 \Rightarrow \frac{\log 2}{\log 10 - \log 2} = \frac12 \Rightarrow \frac{\log 2}{1-\log 2} = \frac12 \Rightarrow$ $1 - \log 2 = 2\log 2 \Rightarrow 3\log 2 = 1 \Rightarrow \log 2 = \frac13$ $\log_2 7 = 2.8 \Rightarrow \frac{\log 7}{\log 2} = 2.8 \Rightarrow \frac{\log 7}{\frac13} = 2.8 \Rightarrow \log 7 = \frac13 \times 2.8 = \frac13 \times \frac{28}{10} = \frac{28}{30} = \frac{14}{15}$ $\log_{14} 10 = \frac{\log 10}{\log 14} = \frac{1}{\log 2 + \log 7} = \frac{1}{\frac13 + \frac{14}{15}} = \frac{1}{\frac{19}{15}} = \frac{15}{19}$	۵

$\log_3^5 \times \log_2^3 = \log_2^5 \Rightarrow 1/5 \times 1/6 \Rightarrow \log_2^5 = 2/4$

$\log_{15}^6 = \frac{\log_2^6}{\log_2^{15}} = \frac{\log_2^2+\log_2^3}{\log_2^3+\log_2^5} = \frac{1+1/6}{1/6+2/4} = \frac{2/6}{4} = 0/65$ ۶

$\log_6^{12} = \frac{\log_8^{12}}{\log_8^6}$ $\qquad \log_8^{12} = \log_8^4 + \log_8^3 = \log_{2^3}^{2^2} + \log_{2^3}^3 = \frac23\log_2^2 + \frac13\log_2^3 \Rightarrow$

$\log_8^4 = \log_{2^3}^{2^2} = \frac23 \log_2^2 = \frac23$

$\log_8^{18} = m \Rightarrow \log_8^9 + \log_8^2 = m \Rightarrow \log_{2^3}^{3^2} + \log_{2^3}^2 = m \Rightarrow \frac23\log_2^3 + \frac13\log_2^2 = m \Rightarrow$ ۷

$\frac23\log_2^3 + \frac13 = m$

$\log_2^3 = \frac{3m-1}{2}$

$(0/4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \xrightarrow[\frac{40}{100}=\frac25]{\frac{125}{8}=\left(\frac52\right)^3} \left(\frac25\right)^{2x-1} = \left(\left(\frac52\right)^3\right)^{x^2} \xrightarrow{\frac52=\left(\frac25\right)^{-1}}$

$\left(\frac25\right)^{2x-1} = \left(\frac25\right)^{-3x^2}$ $\qquad 2x-1 = -3x^2 \Rightarrow 3x^2+2x-1=0 \xrightarrow{\Delta=(2)^2-4(3)(-1)=16}$ ۸

$x = \frac{-2-\sqrt{16}}{6} = -1$ $\qquad x = \frac{-2+\sqrt{16}}{6} = \frac13$

$x=-1$ ، مقدار $9x+1$ منفی می‌شود و تعریف نشده می‌شود $\Leftarrow$ $x=\frac13$ ق ق

$\log_8^{9x+1} = \log_8^{(3+1)} = \log_8^4 = \log_{2^3}^{2^2} = \frac23\log_2^2 = \frac23$

$\frac13\log_2^b = \frac23(1+\log_2^3) \to \log_2^b = 2\log_2^6 = \log_2^{36} \Rightarrow$

$\log_{10}^{108-8} = 2$ ۹

$-4ax^2+bx+\frac12 c=0$

$\frac{1}{-\frac{b}{-4a}} = \log 4 \Rightarrow \frac{4a}{b} = \log 4 \Rightarrow \frac ab = \frac{\log 4}{4} = \frac{2\log 2}{4} = \frac{\log 2}{2}$

$\Rightarrow 2a = b+c \Rightarrow 2\frac ab = 1+\frac ca \xrightarrow{\frac ab = \frac{\log 2}{2}} 2\left(\frac{\log 2}{2}\right) = 1+\frac cb \Rightarrow \frac cb = \log 2 - 1$ ۱۰

$\begin{cases} \frac cb = -1+\log 2 \\ \frac ab = \frac{\log 2}{2} \end{cases} \Rightarrow \frac ca = \frac{-1+\log 2}{\frac{\log 2}{2}} \Rightarrow \frac ca = \frac{\log 2 - \log 10}{\frac12\log 2} = \frac{\log\frac{2}{10}}{\frac12\log 2} \Rightarrow \frac{\log\frac15}{\frac12\log 2} = \log_{\sqrt2}\frac15$ $\Rightarrow$ ادامه حل

$\Rightarrow \left(\frac{1}{\sqrt[8]{2}}\right)^{\frac ca} = \left(\frac{1}{\sqrt[8]{2}}\right)^{\log_{\sqrt2}\frac15} = \left(\frac15\right)^{\log_{\sqrt2}\frac{1}{\sqrt[8]{2}}} = \left(\frac15\right)^{\log_{2^{\frac12}} 2^{-\frac18}} = \left(\frac15\right)^{\left(\frac{-\frac18}{\frac12}\right)\log_2^2} \Rightarrow$

$\left(\frac15\right)^{-\frac14} = 5^{\frac14} = \sqrt[4]{5}$