نام و نام خانوادگی ....[illegible]........ پاسخنامه تشریحی تکلیف شماره ۱۱ ...... کلاس ....[illegible]........

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$\log_n m = a \rightarrow \frac{\log m}{\log n} = a$

$b = \log_{mn} m^2 n = \frac{2\log m + \log n}{\log m + \log n} = \frac{2a+1}{a+1} = 1 + \frac{a}{a+1} = 1/\ldots$

$[b] = 1$

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الف) $y = \sqrt{\frac{x}{\log_{\frac{1}{2}} x}} \Rightarrow \log_{\frac{1}{2}} x \neq 0 \Rightarrow \log_{\frac{1}{2}} x = 0 \rightarrow x \neq 1$

$\frac{x}{\log_{\frac{1}{2}} x} \geq 0 \Rightarrow 0 < x < 1$ مثبت است ، $x > 1$ منفی است $\Rightarrow (0,1)$

ب) $\frac{\log_2 (x^2 - x - 2)}{\sqrt{x^2-1}+1} \Rightarrow x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow x < -1$ یا $x > 2$ I

II $x \geq 1 \Rightarrow (-\infty, -1) \cup (2, +\infty)$

$\sqrt{x^2-1}+1 \Rightarrow x^2 - 1 \geq 0 \Rightarrow x \leq -1$

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$\log_y x = \frac{1}{\log_x y}$

$2\log_9 a + \log_a 3 = 2 \Rightarrow \frac{2}{2}\log_3 a + \log_a 3 = 2 \Rightarrow \log_3 a + \log_a 3 = 2 \rightarrow \log_3 a + \frac{1}{\log_3 a} = 2 \Rightarrow$

$(\log_3 a)^2 + 1 = 2(\log_3 a) \quad \log_3 a = t \rightarrow t^2 + 1 = 2t \Rightarrow t^2 - 2t + 1 = 0 \Rightarrow (t-1)^2 = 0 \Rightarrow t = 1$

$\log_3 a = 1 \Rightarrow a = 3$

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$\log_{10} 2 = 0.3 \quad , \quad \log_{10} 3 = 0.48 \quad \rightarrow 2(\log 3)x = 0.18$

$(\log_{10} \frac{5}{2}) x^2 + (\log 9) x - \frac{\log 125}{\log 5 + \log 2}$

$\log 5 + \log 2 = \log \frac{10}{2} = \log 10 - \log 2 + \log 2 \Rightarrow \frac{1 - 0.3 + 1.4}{1.1}$

$\log 5 - \log 2 \Rightarrow \log \frac{10}{4} = \log 10 - \log 2 - \log 2 = 0.4$

$0.4x^2 + 0.8x - 1.1 = 0 \xrightarrow{\times 10} 3x^2 + 8x - 11 = 0 \Rightarrow x^2 + 8x - 33 = 0 \Rightarrow$

$(x+11)(x-3) \Rightarrow x_1 = -\frac{11}{3} \quad x_2 = 1 \Rightarrow x_1 - x_2 = 1 - (\frac{11}{3}) = \frac{14}{3}$

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$\log_5 2 = \frac{1}{2} \Rightarrow \frac{\log 2}{\log 5} = \frac{1}{2} \Rightarrow \frac{\log 2}{\log \frac{10}{2}} = \frac{1}{2} \Rightarrow \frac{\log 2}{\log 10 - \log 2} = \frac{1}{2} \Rightarrow \frac{\log 2}{1 - \log 2} = \frac{1}{2} \Rightarrow$

$1 - \log 2 = 2\log 2 \Rightarrow 3\log 2 = 1 \Rightarrow \log 2 = \frac{1}{3}$

$\log_2 7 = 2.8 \Rightarrow \frac{\log 7}{\log 2} = 2.8 \Rightarrow \frac{\log 7}{\frac{1}{3}} = 2.8 \Rightarrow \log 7 = \frac{1}{3} \times 2.8 = \frac{1}{3} \times \frac{28}{10} = \frac{28}{30} = \frac{14}{15}$

$\log_{14} 10 = \frac{\log 10}{\log 14} = \frac{1}{\log 2 + \log 7} = \frac{1}{\frac{1}{3} + \frac{14}{15}} = \frac{1}{\frac{19}{15}} = \frac{15}{19}$

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$\log_3^5 \times \log_2^3 = \log_2^5 \Rightarrow 1/5 \times 1/6 \Rightarrow \log_2^5 = 2/4$

$\log_{15}^6 = \dfrac{\log_2^6}{\log_2^{15}} = \dfrac{\log_2^2 + \log_2^3}{\log_2^3 + \log_2^5} = \dfrac{1+1/6}{1/6+2/4} = \dfrac{2/6}{4} = 0/65$ ۶

$\log_6^{12} = \dfrac{\log_8^{12}}{\log_8^6}$ $\quad \log_8^{12} = \log_8^4 + \log_8^3 = \log_{2^3}^{2^2} + \log_{2^3}^3 = \frac{2}{3}\log_2^2 + \frac{1}{3}\log_2^3 \Rightarrow$

$\log_8^4 = \log_{2^3}^{2^2} = \frac{2}{3}\log_2^2 = \frac{2}{3}$

$\log_8^{18} = m \Rightarrow \log_8^9 + \log_8^2 = m \Rightarrow \log_{2^3}^{3^2} + \log_{2^3}^2 = m \Rightarrow \frac{2}{3}\log_2^3 + \frac{1}{3}\log_2^2 = m \Rightarrow$ ۷

$\frac{2}{3}\log_2^3 + \frac{1}{3} = m$

$\log_2^3 = \dfrac{3m-1}{2}$ $\quad \dfrac{\lg 18}{\lg 8} \Rightarrow \dfrac{\lg 3}{\lg 2} = \dfrac{3m-1}{2}$ $\quad \lg_4^{12} = 1 + \lg_4^3 = \dfrac{\lg 3}{2\lg 2} = 1 + \frac{1}{2}\left(\dfrac{3m-1}{2}\right) = \dfrac{3m+3}{4}$

$(0/4)^{2x-1} = \left(\dfrac{125}{8}\right)^{x^2} \xrightarrow[\frac{40}{100} = \frac{2}{5}]{\frac{125}{8} = \left(\frac{5}{2}\right)^3} \left(\dfrac{2}{5}\right)^{2x-1} = \left(\left(\dfrac{5}{2}\right)^3\right)^{x^2} \xrightarrow{\frac{5}{2} = \left(\frac{2}{5}\right)^{-1}}$

$\left(\dfrac{2}{5}\right)^{2x-1} = \left(\dfrac{2}{5}\right)^{-3x^2}$ $\quad 2x-1 = -3x^2 \Rightarrow 3x^2 + 2x - 1 = 0 \xrightarrow{\Delta = (2)^2 - 4(3)(-1) = 16}$ ۸

$x = \dfrac{-2-\sqrt{16}}{6} = -1$ $\quad x = \dfrac{-2+\sqrt{16}}{6} = \dfrac{1}{3}$

$x=-1$ مقدار $9x+1$ منفی می‌شود و تعریف نشده می‌شود $\Leftarrow$ $x = \frac{1}{3}$ ق ق

$\log_8^{9x+1} = \log_8^{(3+1)} = \log_8^4 = \log_{2^3}^{2^2} = \frac{2}{3}\log_2^2 = \frac{2}{3}$

$\frac{1}{3}\log_2^b = \frac{2}{3}(1+\log_2^3) \rightarrow \log_2^b = 2\log_2^6 = \log_2^{36} \Rightarrow$

$\log_{10}^{108-8} = 2$ ۹

$-4ax^2 + bx + \frac{1}{2}c = 0$

$\dfrac{1}{\frac{-b}{-4a}} = \log 4 \Rightarrow \dfrac{4a}{b} = \log 4 \Rightarrow \dfrac{a}{b} = \dfrac{\log 4}{4} = \dfrac{2\log 2}{4} = \dfrac{\log 2}{2}$

$\Rightarrow 2a = b + c \Rightarrow 2\dfrac{a}{b} = 1 + \dfrac{c}{a} \xrightarrow{\frac{a}{b} = \frac{\log 2}{2}} 2\left(\dfrac{\log 2}{2}\right) = 1 + \dfrac{c}{b} \Rightarrow \dfrac{c}{b} = \log 2 - 1$ ۱۰

$\begin{cases} \dfrac{c}{b} = -1 + \log 2 \\ \dfrac{a}{b} = \dfrac{\log 2}{2} \end{cases} \Rightarrow \dfrac{c}{a} = \dfrac{-1+\log 2}{\frac{\log 2}{2}} \Rightarrow \dfrac{c}{a} = \dfrac{\log 2 - \log 10}{\frac{1}{2}\log 2} = \dfrac{\log \frac{2}{10}}{\frac{1}{2}\log 2} \Rightarrow \dfrac{\log \frac{1}{5}}{\frac{1}{2}\log 2} = \log_{\sqrt{2}} \frac{1}{5}$ $\xrightarrow{\text{ادامه حل}}$

$\Rightarrow \left(\dfrac{1}{\sqrt[4]{2}}\right)^{\frac{c}{a}} = \left(\dfrac{1}{\sqrt[4]{2}}\right)^{\log_{\sqrt{2}} \frac{1}{5}} = \left(\dfrac{1}{5}\right)^{\log_{\sqrt{2}} \frac{1}{\sqrt[4]{2}}} = \left(\dfrac{1}{5}\right)^{\log_{2^{\frac{1}{2}}} 2^{-\frac{1}{4}}} = \left(\dfrac{1}{5}\right)^{\left(\frac{-\frac{1}{4}}{\frac{1}{2}}\right)\log_2^2} \Rightarrow$

$\left(\dfrac{1}{5}\right)^{-\frac{1}{2}} = 5^{\frac{1}{2}} = \sqrt{5}$