Subject:
Date

لیلا کریمی — ۱/۵ تکلیف شماره ۲۳ — ۲۰

۱) $\log_{mn} m^2 n = b$ $\qquad \log_n m = a \to n^a = m$

$\log_{mn} mn + \log_{mn} m = b \Rightarrow 1 + \log_{n^{a+1}} n^a = b \Rightarrow \frac{a}{a+1} + 1 = b$ ۲

$[b] = [1+\frac{a}{a+1}] = 1 + [\frac{a}{a+1}] = 1$

کوچکتر از یک

۲) الف) $y=\sqrt{\frac{x}{\log_{\frac{1}{2}} x}}$ $\quad$ ① $x>0$ $\quad$ ② $\log_{\frac{1}{2}} x \neq 0 \Rightarrow x \neq 1$

③ $\frac{x}{\log_{\frac{1}{2}} x} \geq 0$ $\quad$ (0, 1: $-\ +\ -$) $\Rightarrow D=(0,1)$ ۲

ب) $y=\frac{\log_4 (x^2-x-2)}{\sqrt{x^2-1}+1}$ $\quad$ ① $x^2-x-2>0 \Rightarrow (x-2)(x+1)>0 \Rightarrow$ (−1, 2: $+\ -\ +$)

② $\sqrt{x^2-1} \neq -1$ ✓ $\quad$ ③ $x^2-1 \geq 0 \Rightarrow x^2 \geq 1 \to x \geq 1 \ , \ x \leq -1$

$\Rightarrow D = (-\infty,-1) \cup (2,+\infty)$

۳) $2\log_9 a + \log_a 3 = 2 \to \underbrace{\log_3 a}_{t} + \log_a 3 = 2 \Rightarrow t+\frac{1}{t}-2=0 \Rightarrow t^2-2t+1=0$

$\Rightarrow \underbrace{(t-1)^2}_{t=1}=0 \to \log_3 a = 1 \Rightarrow a=3$ ۲

۴) $(\log 5 - \log 3)x^2 + (2\log 3)x - (\log 5 + \log 3) = (\log 10 - \log 2 - \log 3)x^2 + (2\log 3)x - (\log 10 - \log 2 + \log 3)$

$= (1-0.3-0.4)x^2 + (2\times 0.4)x - (1-0.3+0.4) = 0.3x^2+0.8x-1.1=0$

$\xrightarrow{\times 10} 3x^2+8x-11=0$ $\quad x=1 \ , \ x=-\frac{11}{3}$ $\quad \Rightarrow 1-(\frac{-11}{3}) = \frac{14}{3}$ ۲

۵) $\log_5 2 = 0.5 \Rightarrow \log_2 5 = 2$ $\qquad \log_{14} 10 = \frac{\log_2 10}{\log_2 14} = \frac{\log_2 5 + \log_2 2}{\log_2 7 + \log_2 2}$

$\log_2 7 = 2.8$

$= \frac{2+1}{2.8+1} = \frac{3}{3.8}$ ۲

PAPCO

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$\log_2 3 = 1.6 \longrightarrow \log_3 2 = \frac{5}{8}$ $\qquad \log_{15} 6 = \frac{\log_3 6}{\log_3 15} = \frac{\log_3 2 + \log_3 3}{\log_3 5 + \log_3 3} = \frac{\frac{5}{8}+1}{1.5+1} = \frac{13}{20}$ (16

$\log_3 5 = 1.5$

$\log_8 18 = \frac{1}{3}(\log_2 9 + \log_2 2) = m \longrightarrow 2\log_2 3 + 1 = 3m \Rightarrow \log_2 3 = \frac{3m-1}{2}$ (17

$\log_4 12 = \log_4 4 + \log_4 3 = 1 + \frac{1}{2}\log_2 3 = 1 + \frac{3m-1}{4} = \frac{3}{4}m + \frac{3}{4}$

$(0.4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \longrightarrow \left(\frac{2}{5}\right)^{2x-1} = \left(\frac{5}{2}\right)^{3x^2} \Rightarrow 3x^2 = 1 - 2x$ (18

$\Rightarrow 3x^2 + 2x - 1 = 0 \rightarrow \frac{1}{3}, \; -1$

① if $x = \frac{1}{3} \rightarrow \log_8^{9x+1} = \log_8 4 = \frac{2}{3}\log_2 2 = \frac{2}{3}$

② if $x = -1 \rightarrow \log_8^{9x+1} = \log_8 -8$ تعریف نشده $\times$

$\log_8 b = \frac{2}{3}(1 + \log_2 3) = \frac{2}{3}\log_2 6 = \log_8 36 \Rightarrow b = 36$ (19

$\log(3b-8) = \log 100 = 2$

جمع ریشه‌ها $= \frac{-b}{-4a} = \frac{b}{4a} \longrightarrow$ حاصلضرب ریشه‌ها $= \frac{4a}{b} = 2\log 2 \Rightarrow 2\frac{a}{b} = \log 2 \rightarrow \frac{b}{a} = \frac{2}{\log 2}$ (10

$a = \frac{b+c}{2} \longrightarrow c = 2a - b$ $\qquad \frac{c}{a} = \frac{2a-b}{a} = 2 - \frac{b}{a} = 2 - \frac{2}{\log 2} = 2 - 2\log_2 10 = 2 - \log_2 100$

$\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}} = (\sqrt[8]{2})^{-(2-2\log_2 10)} = 2^{-\frac{1}{4}(1-\log_2 10)} = 2^{-\frac{1}{4}\log_2 0.2} = 2^{-\frac{1}{4}\log_2 5^{-1}}$

$= 2^{\frac{1}{4}\log_2 5} = 5^{\frac{1}{4}\log_2 2} = 5^{\frac{1}{4}} = \sqrt[4]{5}$