تکلیف شماره ۲۳ آیدا محمدی‌گرگر یازدهم 13 دختر

۱۷٫۲۵

۱)

$\log_n^m = a \ , \ \log_{mn}^{m^2 n} = b$

$a>0 \quad [b]=?$

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$\rightsquigarrow \dfrac{\log_n^{m^2 n}}{\log_n^{mn}} = b = \dfrac{\log_n^{m^2+1}}{\log_n^{m+1}}$

$\dfrac{2(a+1)-1}{a+1} = 2-\dfrac{1}{a+1}$

$\hookrightarrow 1<2-\dfrac{1}{a+1}<2$

$[b]=1$

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۲) الف)

$y=\sqrt{\dfrac{x}{\log_{1/2}^{x}}} \to x>0 \qquad \log_{\frac{1}{2}}^{x} \neq 1 \longrightarrow x \neq 1$

$\log_{\frac{1}{2}}^{4} = -2 \longrightarrow D_f = 0<x<1$

۱٫۵

ب)

$\dfrac{\log_4 (x^2-x-2)}{\sqrt{x^2-1}+1} \qquad$ جواب $\rightarrow D_f = (-\infty,-1] \cup [1,+\infty)$

$(x-2)(x+1)>0$

$\begin{array}{c} \quad -1 \quad 2 \\ + \mid - \mid + \end{array} \quad x^2-1 \geq 0 \longrightarrow x^2 \geq 1 \longrightarrow x \geq 1$ یا $x \leq -1 \longrightarrow$

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۳) $2\log_x^a + \log_a^{\sqrt{x}} = 2 \qquad m+\dfrac{1}{m} = \dfrac{m^2+1}{m} = 2 \longrightarrow m^2-2m+1=0$

$x=9 \quad a=?$

$\hookrightarrow (m-1)^2 \to m-1=$

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$2\log_{3^2}^a + \log_a^3 = 2 \longrightarrow \log_{\frac{3}{m}}^{a} + \log_a^3 = 2$

$\log_3^a$

$a=3$ جواب

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۴) $\log 2 \simeq 0{,}3 \qquad \log 3 \simeq 0{,}4$

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$\left(\log \frac{5}{3}\right) x^2 + (\log 9)x - \log 15 = 0$

$\log \frac{5}{3} = \log 5 - \log 3 = \cancel{\log 10} - \cancel{\log 2} - \cancel{\log 3} = 0{,}3$

$\log 9 = 2\log 3 = 0{,}8$

$\log 15 = \log 5 + \log 3 = \log 10 - \log 2 + \log 3 = 1{,}1 \qquad$ اختلاف $=$

$0{,}3x^2 - 0{,}8x - 1{,}1 = 0 \xrightarrow[\times 10]{} 3x^2 - 8x - 11 = 0$

$\hookrightarrow 7x^2 + 8x - 32 = 0 \rightsquigarrow$ ۱ و $-\frac{1}{3}$

۹

5) $\log_2^7 = 2{,}8$ $\quad \log_{14}^{10} = ?$

$\log_5^2 = 0{,}5 \leadsto \log_2^5 = 2$

$\log_{14}^{10} = \frac{\log_2^{10}}{\log_2^{14}} = \frac{\log_2^5 + \log_2^2}{\log_2^7 + \log_2^2} = \frac{30}{38}$

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6) $\log_3^5 = 1{,}5$ $\quad \log_2^3 = 1{,}6$ $\quad \frac{\frac{10}{16} + \frac{16}{16}}{15 + 10} = \frac{26}{25 \times 16}$

$\log_{15}^6 = ?$ $\quad \times - \log_2^5 = 10$

$\frac{\log_3^6}{\log_3^{15}} = \frac{\log_3^2 + 1}{\log_3^5 + 1} \leadsto \log_2^3 = 16 \to \frac{1}{\log_3^2} = \frac{16}{1}$

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7) $\log_8^{18} = m$ $\quad \log_4^{12} = \frac{1}{2}\log_2^{12} = \frac{1}{2}(\log_2^4 + \log_2^3)$

$\frac{1}{3}$

$\log_2^{18} = \frac{1}{3}(\log_2^2 + 2\log_2^3) \to \frac{1}{3}(1 - 2\log_2^3) \to 3m = 1 + 2\log_2^3 = m$ ۱,۵

$\frac{1}{4} = (2 + \frac{3m-1}{2}) = \frac{1+3m}{4}$ $\quad \log_2^3 = \frac{3m-1}{2}$

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8) $(0{,}4)^{2x-1} = (\frac{125}{8})^{x^2}$ $\quad (\frac{4}{10})^{2x-1} = (\frac{5}{2})^{3x^2} \to \frac{2^{2x-1}}{5^{2x-1}} = \frac{5^{3x^2}}{2^{3x^2}} \leadsto 2^{3x^2+2x-1} = 5$

$\log_8 (9x+1) = ?$ $\quad 3x^2 + 2x - 1 = 0 \to m^2 + 2x - 3 = 0 \to x = \frac{1}{3}, \; x = -1$ غ ق ق

$\log_8^{9x+1} = \log_8^4 = \frac{2}{3}$

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9) $\log_8^b = \frac{2}{3}(1+a)$, $\quad \log_2^3 = a$ $\quad \log_{10}(3 \times 36 - 8) = 2$

$\log(3b - 8) = ?$ $\quad \frac{1}{3}\log_2^b = \frac{2}{3}(1+a) \to \log_2^b = 2\log_2^6 \to b = 36$

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10) $-4ax^2 + bx + \frac{1}{2}c = 0$ $\quad S = \frac{b}{4a} \to \frac{1}{S} = \frac{4a}{b}$, $2a = \frac{b+c}{2} \leadsto 2a = b + c \to$

$\frac{b+c}{a} = 2$

$(\frac{1}{\sqrt[3]{2}})^{c/a}$ $\quad 4\log 2 = \frac{4a}{b} \to \log_2^1 = \frac{2a}{b} \leadsto \frac{1}{2} \times \frac{1}{\log_2^{10} = \frac{a}{b}}$ ۱,۷۵

$2\log_2^{10} = \frac{b}{a}$, $\frac{c}{a} = 2 - 2\log_2^{10} = -2\log_2^5 = \frac{c}{a} = 2 - \log_2^{10}$

$[\frac{1}{\sqrt{2}}]^{c/a} = 2^{-\frac{1}{8} \times -2\log_2^5} = 2^{\log_2 \sqrt[4]{5}} = \cancel{\frac{1}{\sqrt{5}}}$ $\quad \sqrt[4]{5}$

۲ ب)

$\sqrt{x^2-1} \to (-\infty,-1]\cup[1,+\infty)$ ①

$\log_4^{x^2-x-2} \to x^2-x-2>0 \to (-\infty,-1)\cup(2,+\infty)$ ②

$\xrightarrow{①\cap②} (-\infty,1)\cup(2,+\infty)$

۴)

$\log 5 = 0.7 \qquad \log 6 = 0.8 \qquad \log 15 = 1.1 \qquad \log\frac{5}{3} = 0.3$

$\to 0.3x^2+0.8x-1.1 \to \begin{cases} x_1 = 1 \\ x_2 = \dfrac{-1.1}{0.3} \end{cases} \to x_1 - x_2 = \dfrac{14}{3}$

$$\frac{\log_3 6}{\log_3 15} = \frac{\log_3 2 + \log_3 3}{\log_3 5 + \log_3 3} = \frac{\frac{1}{1.6}+1}{1.5+1} = \frac{13}{20} = 0.65$$ ۳)

$\dfrac{\log 18}{\log 8} \Rightarrow \dfrac{\log 3}{\log 2} = \dfrac{3m-1}{2} \qquad \log_4 12 = 1 + \cancel{\log_4 3} \nearrow \dfrac{\log 3}{2\log 2} = 1 + \dfrac{1}{2}\left(\dfrac{3m-1}{2}\right) = \dfrac{3m+3}{4}$ ۷)