تست تشریحی ۱

$$\log_{mn}^{m^2 n} = \frac{\log_n^{m^2 n}}{\log_n^{mn}} = \frac{2\log_n^m + 1}{\log_n^m + 1} = \frac{2a+1}{a+1} = 1 + \frac{a}{a+1} \to <0$$

(1

$$\Rightarrow \left[\log_{mn}^{m^2 n}\right] = 1$$

سوال ۲۰ و ۲۱

الف) $y = \sqrt{\frac{x}{\log_{\frac{1}{2}}^x}}$ $\quad \frac{x}{\log_{\frac{1}{2}}^x} \geq 0$ $\quad x>0$

$\log_{\frac{1}{2}}^x \to x>0$ (I)

$\log_{\frac{1}{2}}^x \neq 0 \Rightarrow x \neq 1$

$\log_{\frac{1}{2}}^x > 0 \Rightarrow (0,1)$

$\Leftrightarrow x<1$

ب) $\dfrac{\log^{x^2-x-2}}{\sqrt{x^2-1}+1}$ $\quad (-\infty,-1) \cup (2,+\infty)$ (I)

$x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1)$

	-1		2	
	+	0	−	0	+

$\sqrt{x^2-1}+1 \neq 0 \checkmark \quad x^2-1 > 0$

	-1		1	
	+	0	−	0	+

(II)

$\Rightarrow (2,+\infty) \cap (-\infty,-1) \cup (1,-\infty)$ (I)∩(II)

$(-\infty,-1) \cup [2,+\infty)$

سوال ۳)

$2\log_{\frac{a}{9}}^a + \log_a^{a^3} = 2$ $\quad \underbrace{\log_9^a}_{t} + \frac{1}{\log_a^3}\Big\}t$ $\quad t + \frac{1}{t} = 2 \Rightarrow t^2 - 2t + 1 = 0$

$t = 1$

$\Rightarrow \log_3^a = 1 \Rightarrow a = 3$

سوال ۴)

$$\left(\log_5^{\frac{\sqrt{5}}{2}}\right)x^2 + \left(\log_5^9\right)x - \log_5^{15} = 0 \Rightarrow \frac{1}{10}x^2 + \frac{\Delta}{10}x - 1 = 0$$

$\Downarrow$

$3x^2 + 8x - 11$

$\left(\log_5^{10} - \log_5^2\right)$ $\quad \log^2 + \log^9 \to 0/8$ $\quad \log^2 + \log_{10}^{10} - \log_{10}^2$

$\left(\log_{10}^{10} - \log_{10}^2\right) = 1 - 0/2 - 0/3 = \frac{3}{10}$ $\quad 1/2 - 0/3 = 1/1$

$\frac{\sqrt{\Delta}}{a} = \frac{\sqrt{64+132}}{3} = \frac{14}{3}$

سوال ۵)

$\log_{12}^{10} \xrightarrow{\log_{10}} \frac{1}{\log_{10}^{12}} \xrightarrow{\log_2} \frac{\log_2^{10}}{\log_2^{12}} \Rightarrow \frac{\log_2^5 + 1}{\log_2^3 + 1}$ $\quad \frac{1}{\log_5^2} = \frac{1}{1/2} < 2$

$\Rightarrow \frac{3}{2/8} = \frac{30}{28}$

$\frac{15}{14} < 2$

سوال ۶)

$$\log_{15}^4 = \frac{\log_2^4}{\log_2^{15}} = \frac{1+\log_2^3}{1+\log_2^5} \to \frac{10}{14} = \frac{24}{14} = \frac{12}{7} = \frac{12}{20} = 0/45$$

سوال ۷)

همه عملیات با ... (circled note)

$\log_8 18 = m \qquad \log_4 12 \,? \Rightarrow \frac{1}{2}\log_2 12 \;(12 \to 3\times 4) = \frac{1}{2}(2+\log_2 3)$

$\frac{\log_2 18}{\log_2 8} = \frac{1+2\log_2 3}{3} = m \Rightarrow \frac{3m-1}{2} = \log_2 3$

$\to \frac{1}{2}\left(2+\frac{3m-1}{2}\right) = 1+\frac{3m-1}{4}$

$\boxed{\frac{3m+3}{4}}$ *

سوال ۸)

$(0/4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \qquad 125 = \frac{1}{8}\times 1000 \Rightarrow 2^{(4x-2)} \times 10^{(-2x+1)} = 10^{(3x^2)} \times 2^{(-6x^2)}$

$\frac{125}{8} \downarrow \frac{1000}{64}$

$-2x+1 = 3x^2 \quad , \quad 4x-2 = -6x^2 \Rightarrow 3x^2+2x-1=0 \begin{cases} x=-1 & \text{غ ق ق} \\ x=\frac{1}{3} \end{cases}$

چون در این صورت $9x+1<0$ و این جزو شرایط log نیست!

$\log_8 4 \;(4\to 2^2,\ 8\to 2^3) = \boxed{\frac{2}{3}}$ *

سوال ۹)

$\log_8 b \;(b\to x^2,\ 8\to 2^3) \Rightarrow b = x^2 \qquad \frac{2}{3}(\log_2 x) \;(x\to 2\times 3) \Rightarrow \frac{2}{3}(1+\log_2 3) \Rightarrow x=6 \,,\, b=36$

$\log_{10}(108-8) \;(\to 100) = \boxed{2}$ *

سوال ۱۰)

من برای بدست آوردن $\frac{c}{a}$ از کسر $\frac{b}{4a}$ و خواص واسطه حسابی استفاده کردم

$-4ax^2 + bx + \frac{1}{2}c = 0$

مجموع ریشه‌ها

در صورت سوال باید $\frac{c}{a}$ را به دست بیاوریم پس داریم:

$\frac{-b}{-4a} = \frac{+b}{4a} = \frac{1}{\log_{10} 4} = \log_4 10$

$c = 2a-b \;,\; \frac{b}{4a} = \log_4 10 \Rightarrow \boxed{\frac{c}{4a}} = -\log_4 10 + \frac{1}{2} \Rightarrow \frac{c}{a} = 4\left(-\log_4 10 + \frac{1}{2}\right)$

$\left(2^{-\frac{1}{8}}\right)^{4\left(-\log_4 10 + \frac{1}{2}\right)} \to 4\left(\log_4 \frac{1}{5}\right) \Rightarrow \left(2^{-\frac{1}{2}}\right)^{\left(\log_4 \frac{1}{5}\right)} \;\left(\frac{1}{\sqrt{2}}\right) \Rightarrow \frac{1}{5}^{\log_4 \frac{1}{\sqrt{2}}} \qquad \log_4 \frac{1}{\sqrt{2}} = -\frac{1}{4}$

$\left(\frac{1}{5}\right)^{-\frac{1}{4}} = 5^{\frac{1}{4}} = \boxed{\sqrt[4]{5}}$ *